Contribution of Dense Array Analysis to the Identification and Quantification of Basin-Edge-Induced Waves, Part I: Methodology

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Abstract Recent earthquakes have shown that edge-generated surface waves can significantly contribute to increased damages. Most observations of edge-generated surface waves are concerning long-period surface waves propagating in large-size valleys. Since travel times of such waves between valley edges can reach several tens of seconds, they are quite easy to isolate. In small-size structures, reverberating wave trains are mixed and very dense array analysis is required for the identification of basin-induced surface wave trains. The city of Grenoble (French Alps) is located in a small-size deep alluvial valley and faces important site effects (Lebrun et al., 2001). In order to identify and quantify multidimensional site effects in this basin, a very dense array of 29 three-component seismometers over a 1-km aperture was installed within the city. The wave-field complexity as well as the in situ noise characteristics (colored/correlated noise and low signal-to-noise ratio) led us to develop a procedure based on time–frequency coherence of signal waveforms and the multiple signal classification (MUSIC) (Schmidt, 1981) algorithm to identify the main energetic contributions crossing the array. Next, the nature and energy of waves were estimated using some properties of the analytical three-component covariance matrix. Careful methodological investigations were performed in order to better understand and quantify the effects of site constraints on the estimation of wave parameters with the MUSIC technique. Simulations outline the ability of array antennas first to handle difficult scenarios involving multiple, nonstationary, and correlated propagating phases and second to estimate the polarization and energy of waves. The velocity estimation is shown to be much more unstable than backazimuth estimation, and a low signal-to-noise ratio introduces some variation in estimates. Finally, considering the very large number of identified waves, a statistical view of final estimates is suggested for improving the reliability of analysis. In an accompanying article (Cornou et al., 2003), we use this method to investigate the entire wave field of seismic events recorded by the array in order to isolate basin-induced waves.

Introduction

It is widely recognized that the effects of surface geology can significantly contribute to increased damages, as illustrated by recent earthquakes (1985 Mexico; 1989 Loma Prieta; 1995 Kobe; and 1999 Izmit). The impedance contrast between the bedrock and sediment layers (1D site effects) is first of all involved in explaining the amplification of ground motion. Since the middle of the 1980s, however, observational and numerical studies have emphasized the importance of edge-generated surface waves leading to increased amplifications and durations. Such 2D or 3D basin structure effects on wave propagation are largely illustrated through numerical studies (e.g., Horike et al., 1990; Graves, 1993; Hisada et al., 1993; Olsen and Archuleta, 1996; Moczo et al., 1999), but these simulations are still limited to low frequencies (<2 Hz) that are often below frequencies of interest for earthquake engineering purposes. Most observations of locally generated surface waves (Kagawa et al., 1992; Kino-shita et al., 1992; Phillips et al., 1993; Frankel, 1994; Hattamaya et al., 1995; Field, 1996; Graves, 1998; Sato et al., 1999; Satoh et al., 2001) are concerned with long-period surface waves (from 1 to 6 sec) propagating in large-size valleys, and it is thus quite easy to isolate edge-generated surface waves on late-phase arrivals. When structures are of much smaller extent, however, reverberating wave trains are mixed and it becomes difficult to identify locally generated
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Figure 1. Digital elevation map of the Grenoble area and location of the dense array and deep borehole. Example of records (north–south component) on rock and sediment for an $M_l$ 3.5 event occurred 15 km south of Grenoble center (11 January 1999).

surface waves. Nevertheless, such surface waves in a small basin were directly observed at the Euroseistest site near Thessaloniki (Riepl, 1997), in Caille Valley in the French Alps (Gaffet et al., 1998), in Colfiorito in central Italy (Caserta et al., 1998; Rovelli et al., 2001), and in Parkway in New Zealand (Chavez-Garcia et al., 1999). Caserta et al. (1998), Gaffet et al. (1998), Chavez-Garcia et al. (1999), and Rovelli et al. (2001) used very dense array analysis, which is, for the time being, the most appropriate tool to directly identify diffracted waves in such structures.

The city of Grenoble is located on a small-size, deep sediment-filled basin typical of almost all alpine valleys. Previous site effect studies conducted by Lebrun et al. (2001) showed a large amplification (up to 10 times the reference rock site motion) in the frequency band from 0.2 to 5 Hz and an important increase of ground-motion duration throughout the basin for moderate-size seismic events. Such strong site effects are obvious on Figure 1 when comparing, for an $M_l$ 3.5 event that occurred 15 km from Grenoble, motion recorded on the outcropping rock and one downtown on postglacial deposits. Numerical simulations of ground motion within the basin (Cotton et al., 1998; Bard et al., 1999) outlined important trapping effects within the basin and multiple diffraction by sharp borders. Even if these simulations were limited to frequencies below 1.6 Hz, we can suspect important basin-edge-induced waves at higher frequencies. The aim of our study (this article and Cornou et al. [2003] [article 2]) is thus first to identify and characterize the diffracted wave field and second to try to quantify its importance.

Most array processing techniques used in seismology, such as Burg’s maximum entropy (Burg, 1967), Capon’s conventional and high-resolution (Capon, 1969), and the multiple signal classification (MUSIC) (Schmidt, 1981, 1986) methods, are based on the assumption of stationarity of waves, homogeneity of the medium, and spatial whiteness of noise. One generally assumes that seismic waves are stationary on short time observation windows and disregards noise patterns because analysis is most often focused on waveforms having a high signal-to-noise ratio (SNR). In the Grenoble basin, however, industrial noise and moderate seismicity lead to a quite low SNR, and the noise is strongly colored with dominant frequencies around 0.3 and 3 Hz (Lebrun et al., 2001) that are enclosed in the frequency band of the seismic signal. This article is thus dedicated to the adaptation and test of array analysis techniques that handle both the complexity of seismograms (mixing wave trains) and the inherent site constraints, such as low SNR.

First, the general framework of the Grenoble experiment (geophysical settings and inherent site constraint) and the field experiment are depicted. Then, we discuss the inherent site constraints that lead us to prefer the MUSIC algorithm instead of other $f$-$k$ analyses. The basic MUSIC algorithm is described and the processing strategy is drawn as follows:

- The time–frequency domains where signals are the most coherent across the whole array are determined.
anomaly analysis of 10 years of gravity measurements (Vallon, 1999) provided the substratum topography (Fig. 2a) that fits at the borehole location with the substratum depth evaluated through seismic and borehole measurements (Lemeille et al., 2000; Dietrich et al., 2001; Cornou, 2002). Logging operations performed in the borehole, as well as the investigation of contribution of vertical and offset seismic profiles, show that the \( P \)-wave velocity varies between 1500 and 2150 m/sec and the \( S \)-wave velocity between 250 and 950 m/sec from the surface down to the bedrock (Dietrich et al., 2001; Cornou, 2002). \textit{In situ} borehole measurements at 550-m depth have indicated a \( P \)-wave velocity of 4500 m/sec within the bedrock, whereas the seismic refraction profile has provided a \( P \)-wave propagating at 5600 m/sec at the top of the substratum medium (Cornou, 2002). In the following we will consider a \( P \)-wave velocity of 5600 m/sec in the substratum that leads to an \( S \)-wave velocity of 3200 m/sec assuming a Poisson coefficient of 0.25 in the substratum. Seismic velocity profiles are displayed in Figure 2b. The \( S \)-wave velocity profile exhibits a low velocity value (250 m/sec) within a 40-m-thick surficial layer that leads to an \( S \)-wave velocity contrast of about 2 with the layer underneath. This relatively low \( S \)-wave topmost layer was very recently confirmed by a second borehole drilled near the previous one: cuttings have thus shown a major sand/marl contrast at 40-m depth (Lemeille, 2002). Besides these measurements, Lebrun et al. (2001) conducted a horizontal/vertical (H/V) microzonation study within the area of Grenoble. They observed a resonance frequency of 0.3 Hz in the central part of the basin and another one, in some parts of the city, near 3 Hz that they assigned to a very surficial layer. The spatial distribution over the whole basin of fundamental frequency values agrees with the gravimetric model when assuming a mean \( S \)-wave velocity of about 700 m/sec (Lebrun, 1997). Moreover, attenuation values have been measured within the frequency range 15–40 Hz and were found to be rather stable over the sediment thickness with a quality factor of 40 and 20 for \( P \)- and \( S \)-seismic waves, respectively (Cornou, 2002).

Site Constraints and Array Antenna Type

In our study, the choice of array analysis type has to cope with the following constraints: (1) moderate seismicity and a high level of industrial noise produces a low SNR;
(2) the background noise is colored since it exhibits two dominant frequencies, around 0.3 and 3 Hz (Lebrun, 1997); and (3) correlated waves (i.e., waves with different azimuths but the same frequency and phase content crossing the array at the same time) might give inaccurate wave parameter estimates. The first constraint requires an array technique as in sensitive as possible to SNR. Although the performance of any array technique deteriorates in the case of low SNR, the MUSIC algorithm is nevertheless more robust than Capon’s high-resolution method (Goldstein and Archuleta, 1987). The second constraint may be overcome if the spatial covariance structure of the noise is known (Marcos, 1998). However, the estimation of the noise covariance matrix is difficult in seismological data, particularly since noise is not stationary in time. None of the aforementioned array techniques (MUSIC and $f$-$k$ ones) is able to solve this problem. The last constraint is the one that made us prefer the MUSIC technique. This method can indeed at once solve closely spaced signals and handle difficult scenarios involving highly correlated waves when applying an additional spatial smoothing technique (Goldstein and Archuleta, 1987; Zerva and Zhang, 1996). All comparative studies between Capon’s conventional and high-resolution method and the MUSIC one that have been performed up to now always outlined the better resolving power of MUSIC in the case of multiple, closely spaced arrivals and nonstationary waves (Goldstein and Archuleta, 1987, 1991; Krim and Viberg, 1996; Zerva and Zhang, 1996; Almendros et al., 2000).

Field Experiment and Array Response

1999 Experiment: Array Design and Acquisition Parameters

According to our knowledge of the frequency band amplified within the basin and $P$- and $S$-wave mean velocities, we designed an array large enough to identify waves at the lowest frequencies (at 0.3 Hz, the wavelength is 7000 m for a wave propagation velocity of 2100 m/sec) and small enough to follow wave trains propagating at the highest frequencies (wavelengths down to 30 m for a 10-Hz wave propagating at 300 m/sec). The final array design and location were mainly constrained by the kind and number of available sensors and by the urban context facilities. Thus, 29 three-component seismic stations were installed in the eastern part of the city, as indicated on Figure 1. The bedrock topography derived from gravimetric data displays a dipping bedrock interface straight below the array, as illustrated in Figure 3. Figure 4 shows the array geometry with sensors arranged in concentric rings: 16 L22 MarkProducts sensors (with a flat response between 2 and 50 Hz) were located in two inner rings with an 80-m aperture; 12 wider band ones (3 Lennartz Le3D-5s and 9 Guralp CMG40-20s, with a flat response from 0.2 and 0.05, respectively, to 50 Hz) were installed in a maximum 1-km aperture outer ring; and 1 CMG40-20s sensor was added in the center of the array. Sensor locations were precisely determined using static Global Positioning System (GPS) measurements (precision of about 0.3 m). Sensors were connected either to Reftek-72A2 or to a Minititan-3XT recorder. These instruments are part of the portable Lithoscope and the French mobile accelerometric (RAM) networks (www.lgit.obs.ujf-grenoble.fr). Data were continuously recorded, time synchronization was provided by continuous GPS receivers (time accuracy less than 1 msec), and the sampling rate was fixed to 125 Hz on each channel. As explained in article 2, the outer array with wideband sensors was dedicated for investigating frequencies below 1 Hz, while the inner array with L22 and the central CMG40 sensor was dedicated to studies above 1 Hz.

Array Resolution and Aliasing Phenomena

The capacity of an array to identify crossing wave trains is first of all constrained by the characteristics of its transfer function in the wavenumber domain ($k_x$, $k_y$). The spatial distribution of sensors sets a spatial Nyquist wavenumber $k_{max}$. In case of a linear equispaced array, this wavenumber is $2\pi/2\Delta x$ with $\Delta x$ the spatial sampling. But for less simple array geometries, this Nyquist wavenumber is much more related to the maximal offset $\Delta x_{max}$ between adjacent sensors, as illustrated in Gaffet et al. (1998). Figure 5a displays the transfer function of the inner and outer arrays. For the inner array, $\Delta x_{max}$ equals 30.6 m, which leads to $k_{max} \approx 0.1$ rad/m. For the outer array, it is not so easy to evaluate $\Delta x_{max}$ from the array geometry. We also used coordinates of the first aliasing peak present on the transfer function to determine the Nyquist wavenumber. Coordinates of this peak are (0.01, 0.01) rad/m and lead to $k_{max} = 0.007$ rad/m. From values of $k_{max}$, we deduce at each frequency the minimum value of apparent velocity the array will be able to solve without ambiguity. Figure 5b shows for each array the minimum values of apparent velocities we will look at in this study. As shown by Gaffet et al. (1998), the use of this Nyquist wavenumber underestimates the real resolution of the array. But in the case of blind array analysis of signals composed of numerous propagating phases, the restriction introduced on the value of $k_{max}$ will certainly reduce errors of wave-train evaluation characteristics.

MUSIC Algorithm

The MUSIC algorithm was first developed by Schmidt (1981, 1986) and then adapted to seismic purposes by Goldstein and Archuleta (1987) to identify direction and apparent velocities of the main energetic contributions crossing an array. We briefly describe here the method in case of stationary waves and spatially white additive noise. Further details may be found in Goldstein and Archuleta (1987, 1991) or in Marcos (1998). Suppose that a set of $q$ ($\ll N$) plane waves with an angular frequency $\omega$ are propagating in a homogeneous medium across an array of $N$ sensors. The signal received at station $\mathbf{r}$, at time $t$, $\psi(\mathbf{r}, t)$, is given by
\[ R_{ij} = \sum_{n=1}^{q} |A_n|^2 e^{i(k_n x_1 - \omega t + \phi_n(t))} + \eta^2 \delta_{ij} \]  

and the covariance matrix can be written as

\[ R = USU^* + \eta^2 I, \] 

where

\[ U = (\vec{u}(k_1), \vec{u}(k_2), \ldots, \vec{u}(k_q)), \]

\[ \vec{u}(k_m) = [e^{i\vec{k}_m \cdot \vec{x}_1}, \ldots, e^{i\vec{k}_m \cdot \vec{x}_N}]^T, \]

\[ S = \text{diag}(|A_1|^2, |A_2|^2, \ldots, |A_q|^2, 0, \ldots, 0), \] 

\[ I \] is the identity matrix, and \( \eta^2 \) the intensity of the noise. MUSIC uses the fact that the eigenstructure of \( R \) consists of a signal subspace determined by the \( q \) larger eigenvalues and a noise subspace determined by the \( N - q \) smaller eigenvalues. Using the orthogonality property between signal and noise subspaces, the signal direction vectors are evaluated by searching the signal vectors \( \vec{a}(\vec{k}) \) that have minimal projection in the noise subspace. This is equivalent to finding the peaks of the directional function (MUSIC spectrum)

\[ D(\vec{k}) = \frac{1}{\vec{a}(\vec{k})^T \cdot \vec{E}_n}, \] 

where

- \( a(\vec{k}) \) denotes the hermitian conjugate and \( \langle \rangle \), the average on time. When the \( q \) signals are stationary, not correlated, and the noise is spatially white, elements of the covariance matrix are defined as

\[ \psi(\vec{x}_i, t) = \sum_{m=1}^{q} A_m e^{i(k_m \cdot \vec{x}_i - \omega t + \phi_m(t))} + \eta(\vec{x}_i, t), \] 

where \( A_m \) is the amplitude, \( k_m \) the wavenumber, \( \phi_m(t) \) the phase of the \( m \)th signal, and \( \eta(\vec{x}_i, t) \) the noise. Elements of the covariance matrix are defined as

\[ R_{ij} = \langle \psi(\vec{x}_i, t) \otimes \psi(\vec{x}_j, t) \rangle, \] 

with \( \langle \rangle \) the average on time. When the \( q \) signals are stationary, not correlated, and the noise is spatially white, elements of the covariance matrix are defined as

\[ R_{ij} = \sum_{n=1}^{q} |A_n|^2 e^{i(k_n \cdot \vec{x}_i - \omega t + \phi_n(t))} + \eta^2 \delta_{ij}. \]
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Figure 5. (a) Outer and inner array transfer function. (b) Investigation limits for phase velocity for inner and outer array.

where \( \mathbf{E}_q \) is the matrix formed with eigenvectors associated to the \( N - q \) small eigenvalues of \( \mathbf{R} \) and \( \mathbf{\bar{d}}(\mathbf{\bar{k}}) = [e^{i\mathbf{k} \cdot \mathbf{x}_1}, \ldots, e^{i\mathbf{k} \cdot \mathbf{x}_n}] \). An estimate of the signal power is recovered through inversion from the eigenstructure formulation of the covariance matrix, as detailed in Goldstein and Archuleta (1987). Location of a peak on the MUSIC spectrum provides the azimuth of the signal from the array (back-azimuth), and its apparent propagation velocity \( v \) is given by the relation \( v = \frac{o}{k} \), where \( o \) is the frequency where signal power is maximum. Cross-spectral matrices were evaluated at various frequencies with a frequency channel sampling of 0.01 and 0.1 Hz for studies below and above 1 Hz, respectively.

Data Processing

Array Processing Strategy

MUSIC as well as other array techniques basically corresponds to a time-delay estimation problem: the time delays are simply seen as phase shifts on the nondiagonal terms of the covariance matrix under the assumption of narrowband signal. Since, in practice, phase shifts are never null whatever the time-window length and signal type used, the MUSIC spectrum always exhibits peaks that could be associated to waves having physically acceptable properties (velocity) even if there is no real wave traveling across the array. These pseudo-waves can be eliminated by applying a threshold on the signal energy finally evaluated. In that case, however, we risk eliminating real but not energetic signals. Thus, we preferred to apply this algorithm to the most coherent part of the seismograms to ensure the physical existence of our results. We first explain how we selected the time–frequency windows used in the array analysis. Next, the MUSIC algorithm limitations are discussed in close relation with Grenoble site constraints. Finally, wave polarization properties are used to characterize wave types and to evaluate the energy carried by each identified wave.

Window Selection: Coherence Analysis

We first compute the coherence \( C_i(t, f) \) over time \( t \) and frequency \( f \) for every pair \( i \) of array stations (\( N \) sensors) by using a running window technique (Poupinet et al., 1984):

- For the inner array, coherence analysis was performed between 1 and 15 Hz by using a running time window length of 1.5 sec delayed by 10 times the sampling rate \( dt = 0.008 \) sec.
- For the outer array, coherence analysis was performed between 0.1 and 1 Hz by using a running time window length of 15 sec delayed by 10 times the sampling rate \( dt = 0.128 \) sec, after decimation of the original 0.008-sec sampling rate.

We thus get \( N(N - 1)/2 \) time–frequency coherence estimates having a time sampling \( \Delta T \) of 0.08 sec for the inner
array and 1.28 sec for the outer array. Then, coherences were simply averaged to get a mean across the array:

$$C(t, f) = \frac{2}{N(N-1)} \sum_{j=1}^{N(N-1)/2} C(t, f).$$

(9)

This averaging has the main effect of reducing spatially nonstationary coherent contributions. For the array analysis to be significant, the coherence has to be good enough, that is, larger than a coherence threshold $C_0$. The corresponding $t - f$ windows are determined by finding, at each time increment (delayed by $\Delta T$), all the frequency ranges $(f_1 < f_2)$ where $C(f, t) \geq C_0$ over a time length at least larger than $2/f_1$. This time length corresponds to the minimum duration required by array analysis to give reliable estimates of a wave traveling at a frequency $f_1$. Thus, for a given time $t_i$, let us assume there exist $l_i$ frequency windows $[f_1, f_2]_{j=1, l_i}$

for which such a coherence threshold criterion is fulfilled. Then the total number of windows over the whole signal duration is

$$L(C_0) = \sum_{i=0}^{N_i} l_i(C_0) - \sum_{f_1}^{l_{min}} 2/(f_1 \Delta T).$$

(10)

where $N_i$ is the number of time samples of the $t - f$ coherence map. Obviously, some of the windows are overlapping, and $L(C_0)$ is significantly larger than the exact number of boxes in the $t - f$ domain for which the coherence is larger than $C_0$. However, we built one $C_0$ threshold selection on the dependence of $L$ with $C_0$, as illustrated in Figure 6.

When $C_0$ is zero, the criterion is fulfilled over the whole signal duration within the frequency interval $[f_{min}, f_{max}]$, where $f_{min}$ is the minimum frequency of analysis (1 and 0.1 Hz for the inner and outer array, respectively) and $f_{max}$ is the maximum frequency of analysis (10 and 1 Hz for the inner and outer array, respectively). Then, $l_i = 1$ whatever the time $t_i$, and the number of $t - f$ windows is

$$N_i = \frac{2}{f_{min} \Delta T}.$$  

At the other extreme, when $C_0$ is 1, there is no window satisfying the criterion unless the signals are exactly identical at all sensors.

When $C_0$ increases, noncoherent $t - f$ domains appear and introduce discontinuities in the initial $t - f$ windows pavement so that $L(C_0)$ decreases. After the number of $t - f$ windows reaches a minimum, the higher $C_0$ is, the larger is the number of $t - f$ windows, until it reaches a maximum $L_{max}$ at $C_{max}$. This increase in $t - f$ window number is such that the $|C(t, f) \geq C_0$ over a time length larger than $2/f_1$ criteria is less and less fulfilled when $C_0$ increases. Thus, the previous $t - f$ windows are more and more smashed into several windows. When $C_0$ exceeds the $C_{max}$ value, the coherence threshold is very restrictive and excludes more and more $t - f$ domains: $L$ is monotonously decreasing from $C_{max}$ to $C_0 = 1$. We chose to take a coherence threshold $C_0$ such as $L(C_0) = L_{max}/2$ for frequencies below 1 Hz, and 0.2 times the maximum for frequencies above 1 Hz (Fig. 6). Such a choice allows a homogeneous data processing. In this study, the coherence threshold ranged from 0.9 to 0.98 and the number of time–frequency windows from around 600 to 2200 (see article 2).

**MUSIC Limitations**

**Correlated Signals**

The MUSIC algorithm previously described is ensured to work correctly as long as the signals are not totally correlated to one another, that is, signals with the same frequency are not crossing the array at the same time. In case of strongly correlated signals with additional random noise, however, elements of the covariance matrix $R_{xx}$ can no longer be written as in equation (3) and exhibit an additive signal correlation term (Bokelmann and Baish, 1999)

$$R_{ij} = \sum_{n=1}^{q} |A_n|^2 e^{i\theta_{x,n} x_i - \theta_{x,n} x_j} + \sum_{n=1}^{q} \sum_{m=1}^{q} A_n A_m e^{i\phi_{n,x} x_i - \phi_{n,x} x_j} + \eta^2 \delta_{ij} \text{ (correlation term).}$$

(11)

Thus, in the case of two strongly correlated signals, signals are seen as a blended one that decays the rank of the signal covariance matrix $R$: the singular eigenvalue decomposition of $R$ cannot thus be correctly evaluated. Goldstein and Archuleta (1987) used a subarray spatial averaging method based on the work of Shan et al. (1985) to reduce contributions of signal correlation. However, in their presentation, this technique is a priori restricted to linear and equispaced arrays. Following Bokelmann and Baish (1999), we propose a simple averaging of the various elements of the covariance matrix corresponding to station pairs having similar spacing and azimuth (two-sensor subarrays). This technique may be applied on any arbitrary array geometry (Goncalves, 1999), and its efficiency depends mainly on the redundancy of station pair spacing and azimuth. This spatial smoothing reduces contributions of spatially nonstationary phase shifts induced by correlation phenomena. Without destroying characteristics of signal propagation, this smoothing allows us to recover the eigenstructure that should be observed in the case of partially correlated signals. For the small regular array it was easy to find several pairs with identical distance and azimuth, but in the case of the outer array we had to introduce a tolerance on the similarity in distance and azimuth of subarrays (Bokelmann and Baish, 1999). This tolerance distance (TOL) is defined as
Figure 6. Coherence maps and number of selected coherent windows: an example for the east–west component of a local magnitude 2.5 event (3 February 1999, 16:20:25 UTC) occurring 15 km from Grenoble: (a) Time series and averaged coherence displayed with various coherence thresholds. White dots correspond to the center point of time–frequency windows selected for MUSIC analysis. Only signals recorded at the inner array sensors are used in this analysis. (b) Number of evaluated time–frequency windows as a function of coherence threshold $C_0$. 
\[(\mathbf{\vec{r}_i} - \mathbf{\vec{r}_j}) \cdot \mathbf{\vec{u}} < \text{TOL} \) and \[(\mathbf{\vec{r}_i} - \mathbf{\vec{r}_j}) \cdot \mathbf{\vec{v}} < \text{TOL}, \] (12)

with \(\mathbf{\vec{r}_i}\) and \(\mathbf{\vec{r}_j}\) the \(i\) and \(j\) station pair vectors and \((\mathbf{\vec{u}}, \mathbf{\vec{v}})\) the unit vectors of the Cartesian system. The tolerance distance used for the larger array is 100 m and 2 m for the inner array. By doing so, we allow a deviation of \(20^\circ\) and \(8^\circ\) in azimuth for subarrays having similar \(|\mathbf{\vec{r}}|\) values of about 250 and 14 m for the outer and inner array, respectively. Examples of subarrays used for the outer array are depicted in Figure 7.

**Correlated/Colored Noise.** Bokelmann and Baisch (1999) detailed two other terms from the covariance matrix we need to take into account in case of nonrandom noise: noise covariance and signal–noise correlation terms. In the Grenoble area, the background noise is undoubtedly colored as shown by Lebrun et al. (2001): as its frequency content presents peaks around 0.3 and 3 Hz, the noise is “correlated” with signal. Furthermore, some energy bursts of industrial origin with a narrowband frequency content, ranging between 3 and 5 Hz, appear frequently in noise records. Keeping out energy bursts, effects of this colored noise on the efficiency of array analysis can be qualitatively evaluated by looking at the spatial variability of noise throughout the array. If the background noise in some frequency band is spatially coherent all over the array, the noise and signal–noise correlation term will obviously affect the evaluation of wave properties at such frequencies. Besides, a spatially coherent noise could significantly contaminate the signal when evaluating the most time–frequency coherent windows. The coherence formula used is

\[
C_\theta(f, d) = \frac{|S_{ij}(f, d)|^2}{S_i(f)S_j(f)},
\] (13)

where \(S_{ij}\) is the cross-spectral density of seismograms \(s_i(t)\) and \(s_j(t)\), \(S_i\) and \(S_j\) the power spectral density, \(d\) the distance between two sensors, and \(f\) the frequency. Coherences were simply averaged for similar \(d\) within 2 and 100 m for the inner and outer array, respectively. Coherence was computed using the Welch’s averaged periodogram method. Time series were partitioned into segments of 10-sec length for the largest array (0.1 Hz \(< f < 1\) Hz) and 2-sec length for the smaller one (1 Hz \(< f < 10\) Hz). All time segments were 10% Hanning tapered and overlapped one another by half their length. We calculated both the coherence and the so-called Nakamura (1996) H/V ratio on the most stationary part of a 1-day noise recording. We applied an antitrigger algorithm to eliminate energy bursts.

An example of spatial variability of noise coherence computed on the vertical component of noise records together with the mean H/V ratio observed at the central sensor is displayed in Figure 8. For the small array, coherency decreases drastically over distance. Even if the noise has a 4-Hz spectral signature as shown by the H/V ratio, it has a correlated distance (i.e., distance over which signals are coherent) significantly shorter than that of a signal traveling across the array for which the coherence will remain at the same level over the distance. It means first that, with the method used to select time–frequency windows (threshold \(C_\theta\)) no noise will be identified as a signal one. Secondly, we can expect that signal–noise correlation terms will be reduced through spatial averaging because of their spatial non-stationarity. The only exception at high frequencies will hap-
pen when energy bursts occur during the ground shaking. For the large array however, this decrease of coherence is not so obvious for frequencies below 0.5 Hz and, in the case of very low SNR, we cannot exclude that noise could be mixed with signal.

Estimation of the Number of Signals

The MUSIC algorithm requires that the number of incident signals be known before separating the observed covariance matrix $R$ in signal and noise subspaces. In the case of stationary noncorrelated waves, the number of sources, or rank of the signal subspace, is given by the number of largest eigenvalues of the covariance matrix $R$ and is generally evaluated using statistical Akaike information criterion (AIC) or minimum description length (MDL) criteria (Wax and Kailath, 1985). In the case of correlated signals, as previously mentioned, the rank of the cross-spectral matrix $R$ is reduced, leading to an underestimation of the number of actual signals. As AIC or MDL criteria are not robust enough in the case of a nonspatially white noise field (Krim and Viberg, 1996), we preferred to use a detection based on the noise eigenvalue profile modelization (Marcos, 1998), with eigenvalues determined from the spatially smoothed covariance. An illustration of this procedure is given in Figure 9. The noise eigenvalue profiles computed at each frequency bin in the 2- to 6-Hz frequency band from 4-sec-long noise recordings (Fig. 9a) exhibit some variability in their amplitude level depending on the frequency bin considered. However, the eigenvalues follow an exponential decay between $N_1 = 1$ and $N_2 = N/2$ with $N$, the number of sensors, that can be empirically modeled using the following geometric series:

$$
\lambda_i = \lambda_{N_1} r^{i-1} + \lambda_{N_2}, \quad N_1 < i < N_2,
$$

with $\lambda_i$, the $i$th eigenvalue of the spatially smoothed cross-spectral matrix $R$ and the decay ratio $r$. The mean eigenvalue profile over the 2- to 6-Hz frequency band and its corresponding geometric series are displayed in Figure 9a. Once the recorded noise eigenvalue profile is known, the number of signals is given by finding the number of eigenvalues that are significantly different from the noise eigenvalues. An illustration of such rupture is depicted in Figure 9b, where a 3.5-Hz pseudo-frequency Ricker pulse was superimposed on the previous noise. The theoretical Ricker pulse propagation velocity is 2000 m/sec, and its backazimuth equals 45° from north in a clockwise direction. In Figure 9c, we superimposed another 3.5-Hz pseudo-frequency Ricker pulse with a propagation velocity of 800 m/sec and a theoretical backazimuth of 340° N. In that case, two signals are identified on the eigenvalue profile and propagation characteristics of the two Ricker pulses are quite well recovered by MUSIC analysis, although they are correlated. In practice, the decay ratio depends on the time-window length. We thus preferred to compare values of $r$. The number of signals is obtained when the following inequality is no longer respected: $|r^i - r^{i-1}|^2 < K, 1 < i \leq N_2$ with $r^{i-1} = \lambda_i/\lambda_{i-1}$ and $i$ varying in the decreasing order. After some tests on synthetic signals propagating across the array in addition to the actual seismic noise background, $K$ was empirically fixed to 0.15. This high value simply reveals the dispersion of the observed noise eigenvalues from a mean one (Fig. 9a).

Uncertainties

Goldstein and Archuleta (1991) suggested for a linear equispaced array a general estimate of uncertainties based on cumulative errors caused by uncertainty in time and uncertainty of the peak location in the MUSIC spectrum induced by noise shifting:

$$
\delta \left( \frac{1}{V} \right) = \left( \frac{\delta t}{\Delta x/N} \right)^2 + \left( \frac{1 + \text{SNR}}{\text{SNR}2\pi fL/M} \right)^2 1/2, \quad (14)
$$

with $V$, the estimated velocity, $N$ the number of sensors, SNR the signal-to-noise ratio, $M$ the number of time samples in the analysis window, $f$ the frequency, $L$ the array aperture, $\delta t$ the time uncertainty, and $\Delta x$ the spatial sampling. In our study the precision in time is less than 1 msec and, when considering the first term of equation (14), induced errors are negligible (for a given velocity $V$, the velocity uncertainty is about $=5.4 \times 10^{-5} V^2$ and $=1.7 \times 10^{-5} V^2$, respectively, for the inner and outer array by taking a mean spatial sampling of 20 and 250 m, a number of sensors of 17 and 13, and $\delta t = 1$ msec). The second term of equation (14) is not easy to evaluate since determination of the SNR in each frequency band is not obvious. Furthermore, Krim and Viberg (1996) as well as Marcos (1998) outlined the difficulty in quantifying errors induced by the combination of noise characteristics, sensor calibration, nonzero bandwidth signal, sensor position, and so on. Thus, we preferred to have a numerical view from the uncertainty on peak location uncertainty in the wavenumber domain $(k_x, k_y)$ and of the angular frequency channel sampling $\Delta \omega$ when evaluating the cross-spectral matrix:

$$
\Delta V = V \left( \frac{\Delta k}{k_x^2 + k_y^2} + \frac{\Delta \omega}{\omega} \right), \quad (15)
$$

$$
\Delta \theta = \cos \theta \sin \theta \left( \frac{\Delta k}{k_x} + \frac{\Delta k}{k_y} \right), \quad (16)
$$

with $\theta (\theta = \arctan (k_x, k_y))$ the backazimuth of signal, $V$ the propagation velocity, and $\Delta k$ the wavenumber sampling. $\Delta k$ was fixed to $k_{\text{max}}/400$.

To get an idea of such numerical deviation value ranges, we propagated a [0.1- to 0.45-Hz] sweep across the large array with constant propagation velocities ranging from 500
to 5000 m/sec and a backazimuth of 45°. We added some spatially white noise of very low amplitude (SNR above 100 whatever the considered frequency) and estimated the signal characteristics (velocity and backazimuth) with MUSIC as well as the associated uncertainties calculated with the two previous formulae. Velocity, backazimuth, and their corresponding uncertainties as a function of frequency are depicted on Figure 10. Propagation properties (backazimuth and apparent velocity) are rather correctly estimated by MUSIC whatever the considered frequency, except for very low frequency and high velocity ($V > 3000$ m/sec). The estimated backazimuth uncertainty is quite small whatever the frequency and consistent with the actual deviation. In contrast, estimated velocity uncertainties can be very high at low frequency and larger than the observed differences with respect to the actual value: for a wave propagating at a velocity of 3000 m/sec (star) and with a frequency of 0.2 Hz, the estimated uncertainty is about 500 m/sec. This large value for low-frequency waves propagating at high velocity (that means having a low wavenumber) comes from the definition of $\Delta V$. Reducing the wavenumber sampling will reduce the uncertainty as long as we do not proceed beyond the resolution of the antenna. Considering that (1) this numerical view of uncertainty measures an uncertainty that has obviously to be kept in mind, but does not measure the physical error on wave propagation estimation, and (2) that reducing the wavenumber sampling will drastically increase the time and memory computation requirements, we prefer to keep the wavenumber sampling previously defined. Based on these numerical tests for one wave propagating across the array with a high SNR, we are rather confident that the estimated uncertainties are rather pessimistic upper bounds.
Identifying Additional Characteristics: Type and Energy

As previously mentioned, MUSIC copes with time delays caused by propagation of wavefronts across the array. Thus, a real wave should be identified with the same propagation parameters on each of the three-component of seismograms except in cases of purely vertically incident waves or Love waves. In the latter case, however, the wave should be identified by both the horizontal components except, of course, when Love waves propagate exactly in the direction of one component.

Since MUSIC provides the propagating direction of identified waves, horizontal components can be \textit{a posteriori} rotated into radial and transverse components. One could then use particle motion over rotated components in order to characterize wave-type arrivals. In our case, we expect numerous waves to occur in the same time window, due to multipathing. Therefore, particle motion cannot be used to discriminate the type of arrivals. However, the structure of the covariance matrix on the three-component signal is helpful to evaluate the apparent predominant direction of polarization. Apparent inclination, combined with the distribution of apparent velocities and backazimuths over frequency, should provide enough information to discriminate between body and surface waves, especially to track down Love and \textit{SH} waves. We did not use the strike of the direction of polarization to characterize the wave type because it is much more unstable than the apparent inclination (Vidale, 1986; Jurkevics, 1988). The complex polarization analysis was performed according to Vidale (1986) on analytic three-component seismograms. The covariance matrix is

---

Figure 10. (a) Backazimuth, estimated uncertainties, and actual deviations (from top to bottom) and (b) velocity, estimated uncertainties, and actual deviation (from top to bottom) observed for a 0.1- to 0.45-Hz sweep (top left) propagating across the outer array at various constant velocities indicated by symbols. Velocity and backazimuth uncertainties are calculated using equations (15) and (16). Velocity and backazimuth actual deviation are the deviation from theoretical values.
\[ C(T) = \begin{pmatrix} u(T)u(T)^* & u(T)v(T)^* & u(T)w(T)^* \\ v(T)u(T)^* & v(T)v(T)^* & v(T)w(T)^* \\ w(T)u(T)^* & w(T)v(T)^* & w(T)w(T)^* \end{pmatrix}, \]

where \(^*\) represents complex conjugation, \(u(T), v(T), w(T)\) the analytical transforms of radial, transverse, and vertical motions, and \(T\) the time window. The three eigenvalues of the covariance matrix are \(\lambda_0, \lambda_1, \lambda_2\) (\(\lambda_0 > \lambda_1 > \lambda_2\)) and the eigenvector associated with the largest eigenvalue \(\lambda_0\) is \(\mathbf{x}\). The apparent inclination \(\delta\) from the vertical of the direction of maximum polarization is written as

\[ \delta = 90 - \tan^{-1} \left( \frac{\text{Re}(z_0)}{\sqrt{\text{Re}(x_0)^2 + \text{Re}(y_0)^2}} \right) \quad 0 \leq \theta \leq 90 \]

and the degree of polarization is

\[ P = 1 - \frac{\|\text{Im}(X)\|}{\|\text{Re}(X)\|}, \]

\(P = 1\) for linearly polarized motion and 0 for circular polarization.

From a practical viewpoint and for data we have analyzed in article 2

- time–frequency windows of identified waves are provided by MUSIC
- time signals of individual sensors are windowed using a 10% Hanning taper
- signals are then filtered in frequency around the wave frequency given by MUSIC (the filter used was a one-order Chebychev with a bandwidth of 0.14 \(z\) or 0.7 Hz for frequencies below and above 1 Hz, respectively)
- horizontal components are then rotated into radial and transverse components using the direction of propagation provided by MUSIC array analysis
- covariance matrices at individual sensors are computed and averaged over the whole array to stabilize the polarization estimates (Jurkevics, 1988) (because of the good coherence between waveforms we did not perform any correction for phase propagation before taking the average covariance matrix)
- apparent inclination and apparent degree of polarization are estimated

When multiple waves having a close frequency content are identified in the same time window, this polarization analysis should only give a mean estimation of dominant polarization features. Diagonal elements of the covariance matrix are proportional to the energy carried by radial, transverse, and vertical components. The energy contained in a time–frequency window is first evaluated as

\[ E_{total} = \left( \frac{uu^*}{2} + \frac{vv^*}{2} + \frac{ww^*}{2} \right) dt/3, \]

where \(dt\) is the sampling time and 2 the factor to go back into energy expressed in the time domain.

MUSIC is working around a frequency \(f_0\) (\(f_{\text{min}} < f_0 < f_{\text{max}}\)) and on a time window of length \(T\) \((T = 2f_{\text{min}})\). As the timestep used to select time–frequency windows was fixed to \(\Delta T\) \((< T)\), the same wave (characterized by its main frequency content, backazimuth) would be identified several times through overlapping time windows, leading thus to an overestimation of real energy. Besides, we have to take into account the case of correlated waves. We thus scaled \(E_{total}\) as

\[ E_{i_{total}} = E_{total} \times \frac{P_i}{P_{q_i}} \times \frac{t_i}{T}, \]

with \(E_{i_{total}}\), the energy carried by the wave \(i\), \(E_{total}\) the total energy of the time–frequency window, \(P_i\) the power of wave \(i\), \(P_{q_i}\) the total power of the \(q\) waves \((P_{q_i} = \sum_{i=1}^{q} P_i)\), and \(t_i\) the unoverlapping time delay. When processing the covariance matrix for evaluating energy, we did not filter signals before rotating components and evaluating the covariance matrix since the ratio of energy carried by a wave is given by \(P_i/P_{q_i}\). This way of retrieving energy needs only one description of the wave \((t = T)\) to get the whole energy carried by that wave within the time delay \(T\).

**Test Simulations**

We now present three simulations. The first one illustrates MUSIC’s ability to determine waves in difficult scenarios involving multiple arrivals and correlation phenomena; the second one underlines the bias caused by some particular correlation phenomena. The last one deals with propagation of dispersive surface waves across the array and shows that the MUSIC technique is valid to estimate polarization and energy of waves.

**Multiple Arrivals of Correlated, Nonstationary and Stationary Waves**

We first propagated across the small array a combination of four waves: two correlated pulses (i.e., two transients with the same frequency content and two different azimuths with a controlled SNR) and a nonstationary and a stationary wave. Some spatially white noise was added. Propagation properties as well as the kind of wavelet are listed in Table 1, and Figure 11a displays the signal generated at the center of the array and its theoretical time–frequency representation. In frequency domain, the SNRs are around 2.5 and 7 at 8 and 5 Hz, respectively. We plot in Figure 11b the MUSIC estimated time–frequency representation and in Figure 11c,d the estimated backazimuth and velocity as a function of time.
Table 1

Type and Propagation Parameters of Synthetic Signals

<table>
<thead>
<tr>
<th>Signal Number</th>
<th>Backazimuth (°N)</th>
<th>V (m/sec)</th>
<th>Dominant Frequency (Hz)</th>
<th>Time Occurence (sec)</th>
<th>Type of Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>1000</td>
<td>~5</td>
<td>6–6.5</td>
<td>Gaussian-modulated sinusoidal pulse</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>500</td>
<td>~5</td>
<td>6–6.5</td>
<td>Gaussian-modulated sinusoidal pulse</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>2000</td>
<td>From 2 to 6</td>
<td>5–7.5</td>
<td>Altes signal in time domain</td>
</tr>
<tr>
<td>4</td>
<td>225</td>
<td>1500</td>
<td>8</td>
<td>3.75–8.75</td>
<td>Sinusoidal signal</td>
</tr>
</tbody>
</table>

Figure 11. (a) Synthetic signals generated at the center of the inner array and its theoretical corresponding time–frequency representation; (b) identified signals in the $t - f$ plane; and MUSIC estimates of backazimuth as a function of (c) time and (d) velocity.
and backazimuth, respectively. One can see that MUSIC identified all waves with accurate parameters and an accurate description of the time–frequency distribution.

Nevertheless, one can observe that around 8 Hz, estimated frequencies are dispersed (dispersion around 10%) This scattering is caused by the low SNR of that stationary wave. In order to illustrate such an SNR effect, we also propagated across the array two sinusoidal waves: one is propagating with a frequency of 3 Hz and the other one with a frequency of 7 Hz. Propagation velocity for both waves is 1000 m/sec; the propagation direction of the 3 Hz wave is 0° N and for the 7 Hz wave 90° N. Spatially white noise was added such that the SNR ranged from 0.5 to 50. Figure 12 displays the absolute backazimuth (Fig. 12a), frequency (Fig. 12b), and the velocity spread between the theoretical values and those estimated by MUSIC as a function of SNR (Fig. 12d). One can see that as SNR decreases, the performance of MUSIC also deteriorates since larger spread values are observed at low SNR. The worst estimates, however, are found for the wave propagating at 7 Hz. It can be explained by the higher sensitivity of that high-frequency wave and, for the type of noise added here, to the interference between noise and signal; the signal waveform in the time domain is modified, which leads to a shifted frequency and consequently mislocation of the MUSIC spectrum peaks (Goldstein and Archuleta, 1991). As a consequence, errors on velocity and backazimuth values are larger, especially for the estimated propagation velocity that depends not only on peak location but also on frequency. Figure 12e,f displays the backazimuth and velocity estimated uncertainties as a function of SNR. Backazimuth estimated uncertainties are one decade lower than actual deviation, and velocity estimated uncertainties are lower than actual deviations for low SNR.

When coming back to Figure 11, velocity estimation appears again to be much more unstable than the backazimuth estimation. Larger dispersion is observed for larger propagation velocity. Since velocity standard deviation for these four waves is at most 100 m/sec, we can invoke, as earlier, the effect of noise and frequency shift to explain such features.

Sensitivity of MUSIC to Some Particular Correlation Phenomena

We now consider the case of two correlated waves crossing the array at the same time. Several tests (not all shown here) showed that, if propagation velocities are very different (typically more than 500 m/sec at 0.3 Hz), MUSIC gives accurate estimates of wave parameters; in other cases, however, estimations are much more unstable. We therefore present here only cases of two stationary waves (sinusoids) with identical velocity crossing the outer array. We consider different frequencies (0.3, 0.5, and 0.8 Hz) and identical propagation velocities of 500, 1000, and 2000 m/sec. The direction of one source was fixed at 0° N; the direction of the second one varied from 45° to 315° N with a step of 45°. Spatially white noise was added, leading to an SNR of 10 in the frequency domain. Figure 13 displays estimates of backazimuth, velocity, number of identified waves, and backazimuth and velocity standard deviations as a function of angular spread between the two sources. In the case of small velocity values (v = 500 m/sec), MUSIC detects most often two signals with accurate backazimuth and velocity estimates, except when they propagate in exactly opposite directions (0°–180° N). When the velocity is larger, however, MUSIC identifies either one or two signals. In the latter case, if the angular spread is large enough, the estimate of backazimuth is correct; if not, the identified backazimuth is the mean value of the two source backazimuths. The minimal resolvable angular spread allowing the correct identification of at least one wave was found to be 90° at frequencies of 0.3 and 0.5 Hz and for a velocity of 2000 m/sec. Furthermore, velocities are...
generally largely overestimated compared to the theoretical ones, especially for a theoretical velocity of 2000 m/sec (up to a factor of 2). As is the case for the SNR, estimated uncertainties are lower than the actual deviations.

Thus, correlated waves propagating with close velocities can lead to (1) a nonidentification of one of the waves, (2) a bias on the estimated backazimuth when sources are spatially close together, and (3) a systematic overestimation of velocities in the case of large propagation velocities.

Polarization and Energy

The next simulation is intended to test the ability of MUSIC for estimating wave types and energies for dispersive, polarized waves (a general characteristic of surface waves). By using the velocity model indicated in Table 2, we successively propagate across the outer array three waves: (1) a dispersive Rayleigh wave with a backazimuth of 200° N, (2) a dispersive Love wave with a backazimuth of 240° N, and (3) the two previous Rayleigh and Love waves.
Figure 14. Estimations of backazimuth, phase velocity, apparent inclination, and energy after array processing of (a) a Rayleigh wave propagating across the outer array with a theoretical backazimuth of 200° N; (b) a Love wave propagating across the outer array with a theoretical backazimuth of 240° N; (c) the two previous waves simultaneously propagating across the outer array. Signals observed on the north–south component in case of the single wave (Rayleigh or Love) and of the mixed wave are displayed in (c). Energy of seismogram is the square of the three-component amplitudes scaled by the time sampling $dt$. Surface waves were simulated using the velocity model indicated in Table 2.
waves simultaneously. For each component, MUSIC analysis is performed every time sample of the seismogram ($dt = 0.1 \text{ sec}$) between 0.2 and 1.2 Hz. Figure 14a,b displays results when only one propagating phase (Rayleigh wave or Love wave) is considered. One can see that backazimuth and phase velocity are well estimated. Apparent inclinations are similar to the theoretical ones and the energy of the seismogram is well explained. For the simultaneous combination of the two signals, all propagation parameters and energy are relatively satisfactorily estimated (Fig. 14d). It has to be pointed out, however, that the correlation phenomenon between the Rayleigh and Love phases introduces some scattering on estimates, especially on apparent velocities.

### Toward a Statistical Description of Propagation Characteristic Estimates

We previously mentioned that the method requires a time window long enough to describe at least one wave. Thus, one can easily understand that there is a trade-off between the time-window length and the time separation between arrivals coming from the same backazimuth. Besides, the main drawback of MUSIC outlined by many authors (Krim and Viberg, 1996; Zerva and Zhang, 1996; Marcos, 1998) is the *a priori* assessment of the number of signals in the records. A wrong evaluation of that number can significantly affect MUSIC performance: an underestimation leads to the identification of a blended wave having wrong param-
eters, and an overestimation leads to the detection of a fictitious wave. On the one hand, previous observations pointed out that (1) low SNR produces dispersion on estimates, (2) the velocity estimation is much more unstable than the backazimuth, (3) actual deviation of velocity are expected to be large in some cases, and (4) estimated uncertainties provide lower limits of actual deviation in case of correlated waves or low SNR. On the other hand, for an application in Grenoble, we have to cope with the fact that (1) the SNR is low (typically between 2 and 5), (2) numerous mixed wave trains are expected due to the basin geometry, and (3), the investigation of the entire wave field of seismograms is desired. Therefore, we suggest that estimates of individual waves from MUSIC need to be interpreted carefully and a statistical view of estimates (like the use of histograms) after MUSIC is performed should improve the reliability and interpretation of analysis, especially by emphasizing redundant estimations.

Conclusion

This article, the first in a series of two, was devoted to the presentation of a methodology for the identification of the main energetic contributions of waves crossing a very dense array of seismic sensors on a low SNR environment. The application of MUSIC array analysis has been carefully investigated for better understanding and estimating effects of inherent site constraints (low SNR, colored/correlated noise, correlated signals) on wave parameter estimation. The main methodological outcomes can be summarized as follows:

1. The investigation of the most coherent time-frequency part of seismograms allows us to focus the analysis on the most relevant part of records and to avoid analysis of noise wave trains, since noise has a correlated distance significantly shorter than that of a signal traveling across the array;
2. The spatial smoothing of the cross-spectral matrix allows us to better resolve correlated waves and should reduce effects of correlation between signal and noise;
3. Evaluation of the number of signals in the records is performed using actual noise characteristics;
4. Evaluation of three-component covariance matrix should give enough information on wave polarization characteristics and should give an estimation of the energy carried by each identified wave.

Simulations outlined the ability of MUSIC to handle difficult scenarios involving multiple, nonstationary and correlated waves. Nevertheless, it was shown that estimation is much more unstable for velocity than for the backazimuth and that a low SNR can introduce some dispersion in estimates. Therefore, it was suggested to undertake a statistical analysis of final estimates in order to improve the reliability of interpretation. In the second article, we use this methodology to precisely investigate the entire wave field of 18 seismic events recorded by an array settled in the Grenoble basin in order to isolate and quantify basin-induced surface waves.

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