B002

Making Seismic Data as CSEM Data through the Bäcklund Transform

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SUMMARY

Bäcklund transform is applied to a seismogram in order to construct a new signal where the time is transformed in a pseudo-time the square of its dimension is expressed in seconds. The Bäcklund transform links partial differential equations related to wave propagation to partial differential equations related to diffusion. Potential applications have been performed previously from magnetotelluric data to pseudo-seismic signals by using the inverse of the Bäcklund transform. This transform could be applied to the seismic signal through real computations. Therefore, we propose to transform seismic data into pseudo-CSEM signals for possible applications of these diffusive signals as the initial stage of the full waveform inversion of seismic data.

Introduction

Full waveform inversion allows the reconstruction of physical parameters of the medium from seismic traces recorded mainly at the free surface. This imaging procedure faces difficulties as it is a non-linear inversion with a significant number of secondary minima related to cycle skipping (Tarantola, 1984; Pratt et al, 1998). Different strategies have been proposed to overcome this difficulty by introducing progressively the complexity of the seismic data (Bunks et al, 1995; Brenders and Pratt, 2007; Sears et al, 2008; Brossier et al., 2009) into the optimization engine, by reducing the search zone in the model space or by modifying the objective function (Shin and Min, 2006; Fichtner et al, 2009)

On the data space, Shin and Min (2006) have proposed the use of the Laplace transform in relation with a new objective function related to the logarithm of the seismic signal. Another alternative is the Bäcklund transform (Bragg and Dettman, 1968; Filippi and Frisch, 1969; Filatov, 1984; Lee et al., 1989). This transform, requiring real arithmetic, is an integral operator which transforms the time into a new variable q: it could be applied to a seismic wavefield and the deduced signal verifies a diffusion equation we shall construct. The Bäcklund transform gives a so-called Propagation-to-Diffusion mapping, inverse of the Diffusion-to-Propagation mapping investigated previously by Gibert et al (1994) and Tournerie et al (1995) for magnetotelluric data similar to controlled-source electromagnetism (CSEM) data.

After presenting the Bäcklund transform, we shall illustrate on a given example features of the pseudo-wave field w and we express the interest of this field for mitigating the non-linearity of the inverse problem of fitting seismic wavefields.

Theory

Let us consider the seismic wavefield $u(\vec{x},t)$ at the position $\vec{x}$ verifying the scalar wave equation

$$\Delta u(\vec{x},t) + \frac{1}{c^2(\vec{x})}\frac{\partial^2 u(\vec{x},t)}{\partial^2 t} = 0, \tag{1}$$

where the wave speed is defined by $c(\vec{x})$ and the Laplacian is denoted by Δ. We shall apply the Backlünd transformation defined by the following integral

$$w(\vec{x},q) = \int_0^\infty \frac{t}{2\sqrt{\pi q^3}} e^{-t^2/4q} u(\vec{x},t)dt, \tag{2}$$

where the seismic wavefield $u(\vec{x},t)$ is integrated over time using a diffusive kernel. The new field $w(\vec{x},q)$ turns out to be the solution of the following diffusion equation given by the expression

$$\Delta w(\vec{x},q) + \frac{1}{c^2(\vec{x})}\frac{\partial w(\vec{x},q)}{\partial q} = 0, \tag{3}$$

where the square of the speed acts as a diffuse coefficient.

In the frequency domain, the relation (2) turns out to be

$$w(\vec{x},\omega) = \int_0^\infty e^{-t\sqrt{i\omega}} u(\vec{x},t)dt, \tag{4}$$

while the diffusion equation becomes

$$\Delta w(\vec{x},\omega) + \frac{i\omega}{c^2(\vec{x})} w(\vec{x},\omega) = 0. \tag{5}$$

Knowing that the square of $\sqrt{i}$ is along the diagonal in the plane of the complex frequency (figure 1), we may see that an intermediate kernel could achieve the same computational effort than the Laplace transform with a fast decreasing kernel. The diffusion equation (5) along the diagonal can be used instead of the Helmholtz equation along the real axis or the potential equation along the imaginary axis of the complex frequency plane as the forward problem embedded into the inverse problem formalism.

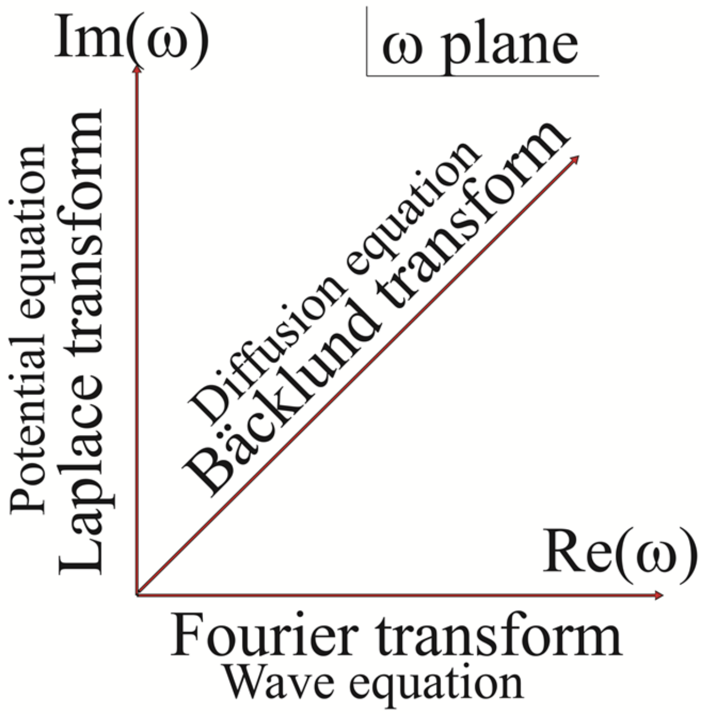

Figure 1: Three directions present specific features of interest for the forward problem. The real axis is related to wave equation, the imaginary axis to the potential equation while the diagonal is linked to diffusion equation.

By doing so, we proceed exactly in the reverse direction as tackled by Virieux et al (1994) where the electromagnetic field recorded at the free surface is transformed into a pseudo propagating wavefield for the application of the asymptotic full waveform inversion

Examples

We perform the modelling of wave propagation using a finite-difference approach of 4th order in space and 2nd order in time for the acoustic wave propagation in a complex model with a salt dome of high velocity contrast as shown by the figure 2. A seismic shot gather is shown in the left panel of the figure 3 and displays complex converted phases related to reflections and diffractions on the salt dome.

The application of the Bäcklund transform for each trace is performed in the time domain using a trapezoidal rule for the integral. On the right panel of the figure 3, one can see the simplification of the complexity with a rapid decrease with the variable q except at the source where the diffusion prevents dissipation. This section can be directly reconstructed using the diffusive equation associated to the wave equation through the Bäcklund transform.

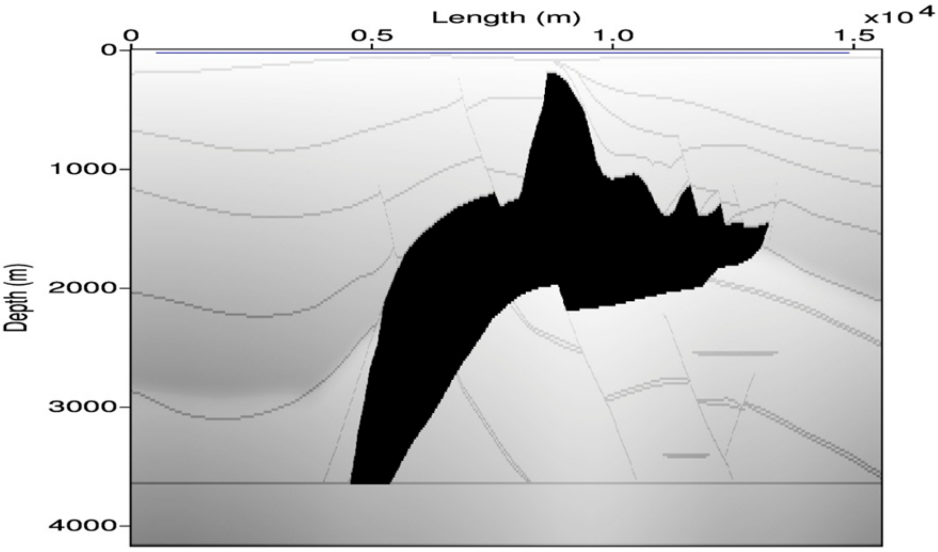

Figure 2 *The salt dome model where an acoustic synthetic seismic acquisition is performed.*

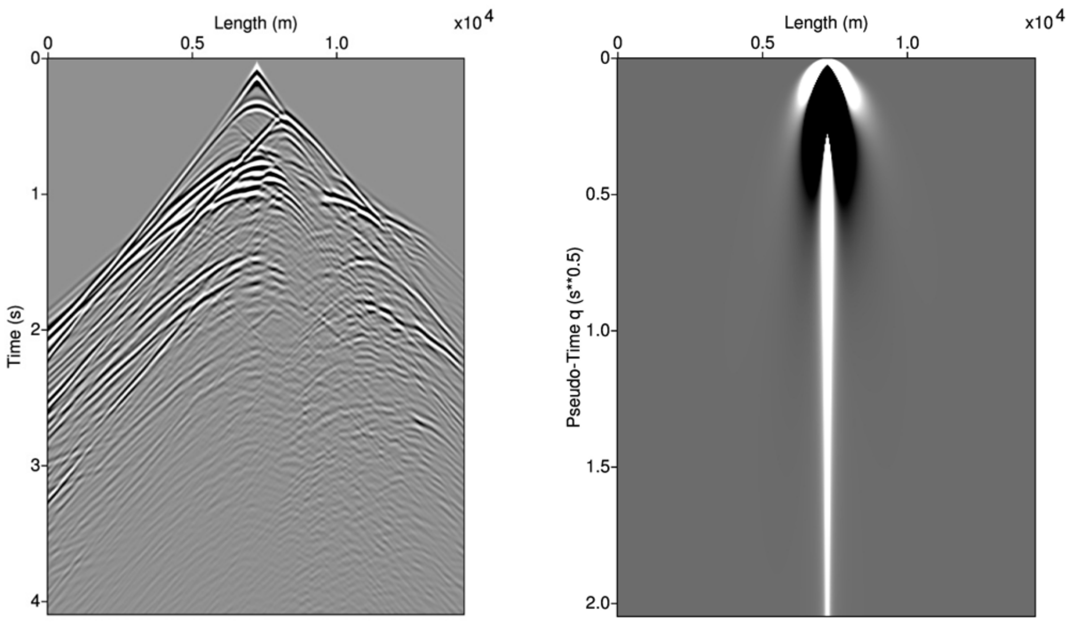

Figure 3 Left panel: seismograms recorded into a complex mode using a finite-difference acoustic wave propagation; right panel: deduced pseudo-diffusive field by the Bäcklund transform. Sensors have been deployed along the profile while the source is in the middle of the acquisition.

These transformed data could be used in an optimization scheme where one considers that the forward problem is governed by the diffusion equation.

Conclusions

The Bäcklund transform allows a significant reduction of the complexity of the seismic signal: the propagation-to-diffusion transformation allows the inverse problem to be less sensitive to secondary minima and, therefore, could be a useful tool for the construction of the initial model for the full waveform inversion, at least its superficial structure. The diffusion is more sensitive to anomalies inside the medium with a rapid decrease related to the skin depth associated to its frequency content: this decrease is weaker than the one related to the Laplace transform and, therefore, better behaviour is expected. Moreover, approaches used for the inversion of CSEM data could be readily applied in this first step procedure of the seismic full waveform inversion.

References

Brenders, A.J. and R.G. Pratt, Efficient waveform tomography for lithospheric imaging: implications for realistic 2D acquisition geometries and low frequency data, Geophysical Journal International, 168, 152-170, 2007.

Bragg, L.R. and J.W. Dettman, Related problems in partial differential equations, Bulletin American Mathematical Society, 74, 375-378, 1968.

Brossier, R., S. Operto and J. Virieux, Seismic imaging of complex onshore strucutres by 2D elastic frequency-domain full-waveform inversion, Geophysics, 74, WCC63-WCC76, 2009.

Bunks, C., F. M. Salek, S. Zaleski and G. Chavent, Multiscale seismic waveform inversion, Geophysics, 60, 1457-1473, 1995.

Fichtner, A., B.L.N. Kennett, H. Igel and H.-P. Bunge, Full seismic waveform tomography for upper-mantle structure in the Australasian region using adjoint methods. Geophysical Journal International, 179, 1703–1725, 2009.

Filatov, V.V., Construction of focusing transformations of non-stationary electromagnetic fields, Geol. I Geofiz. (Soviet Geology and Geophysics), 25, 89-95, 1984.

Filippi, P. and U. Frisch, Relation entre l'équation de la chaleur et l'équation des ondes de Helmholtz, Comptes Rendus Académie des Sciences, Paris, A268, 804-807, 1969.

Gibert, D., B. Tournerie and J. Virieux, Super-resolution electromagnetic imaging of the conductive Earth's interior, Inverse Problems, 10, 341-351, 1994.

Lee, K.H., G. Liu and H.F. Morrison, A new approach to modeling the electromagnetic response of conductive media, Geophysics, 54, 1180-1192, 1989.

Pratt, R. G., C. Shin and G.J. Hicks, Gauss-Newton and full Newton methods in frequency-space seismic waveform inversion, Geophysical Journal International, 133, 341-362, 1998.

Sears, T., S. Singh and P. Barton, Elastic full waveform inversion of multi-component OBC seismic data, Geophysical Prospecting, 56, 843-862, 2008.

Shin, C. and D.-J. Min, Waveform inversion using a logarithmic wavefield, Geophysics, 71, R31-R42, 2006.

Tarantola, A., Inversion of seismic reflection data in the acoustic approximation, Geophysics, 49, 1259-1266.

Tournerie, B., D. Gibert and J. Virieux, Inversion of the COPROD2 magnetotelluric data using a diffusive-to-Propagation mapping (DPM), Geophysical Research Letters, 22, 2187-2190, 1995.

Virieux, J., C. Flores-Luna and D. Gibert, Asymptotic theory for diffusive electromagnetic imaging, Geophysical Journal International, 119, 857-868, 1994.