An overview of the SEISCOPE project on frequency-domain Full Waveform Inversion
Multiparameter inversion and efficient 3D full-waveform inversion

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Abstract

We present an overview of the SEISCOPE project on frequency-domain full waveform inversion (FWI). The two main objectives are the reconstruction of multiple classes of parameters and the 3D acoustic and elastic FWI. The optimization relies on a preconditioned L-BFGS algorithm which provided scaled gradients of the misfit function for each classes of parameter. For onshore applications where body waves and surface waves are jointly inverted, P- and S-wave velocities (V_P and V_S) must be reconstructed simultaneously using a hierarchical inversion algorithm with two nested levels of data preconditioning with respect to frequency and arrival time. Simultaneous inversion of multiple frequencies rather than successive inversions of single frequencies significantly increases the S/N ratio of the models. For offshore applications where V_S can have a minor footprint in the data, a hierarchical approach which first reconstructs V_P in the acoustic approximation from the hydrophone component followed by the joint reconstruction of V_P and V_S from the geophone components can be the approach of choice. Among all the possible minimization criteria, we found that the L_1 norm provides the most robust and easy-to-tune criterion as expected for this norm. In particular, it allowed us to successfully reconstruct V_P and V_S on a realistic synthetic offshore case study, when white noise with outliers has been added to the data. The feasibility of 3D FWI is highly dependent on the efficiency of the seismic modelling. Frequency-domain modelling based on direct solver allows one to tackle small-scale problems involving few millions of unknowns at low frequencies. If the seismic modelling engine embeds expensive source-dependent tasks, source encoding can be used to mitigate the computational burden of multiple-source modelling. However, we have shown the sensitivity of the source encoding to noise in the framework of efficient frequency-domain FWI where a limited number of frequencies is inverted sequentially. Simultaneous inversion of multiple frequencies is required to achieve an acceptable S/N ratio with a reasonable number of FWI iterations. Therefore, time-domain modelling for the estimation of harmonic components of the solution can be the approach of choice for 3D frequency-domain FWI because it allows one to extract an arbitrary number of frequencies at a minimum extra cost.

Introduction

With the tremendous increase of the computational power provided by large-scale distributed-memory platforms and the development of dense 3D multi-component wide-aperture/wide-azimuth surveys, full waveform inversion (FWI) has become a re-emerging technic to build high-resolution velocity models of the subsurface (Virieux and Operto (2009) for a review). Full waveform inversion is a (local) optimization problem, the aim of which is the minimization of the misfit between the recorded and modeled seismic wavefields (Tarantola, 1987). The seismic wavefield is modeled by the full resolution of the two-way wave equation and the inverse problem is solved by local optimization approach, where the gradient of the misfit function can be efficiently computed by the adjoint-state method (Plessix, 2006). While FWI has been originally developed in the time domain in the eighties, Pratt and co-workers have developed efficient FWI in the frequency domain for wide-aperture acquisitions such as cross-hole or dense refraction experiments (Pratt et al., 1998): taking advantage of the redundant control of frequency and aperture angle on the wavenumber coverage in the model space, the inversion can be theoretically limited to few discrete frequencies (Sirgue and Pratt, 2004). These frequencies can be inverted sequentially to design a hierarchical multiscale imaging which is useful to mitigate the non linearity of the inversion. The 2D forward problem for multiple sources is efficiently performed in the frequency domain using direct solvers to solve the linear system resulting from the discretization of the time-harmonic wave equation (Marfurt, 1984). Straightforward implementation of attenuation in frequency-domain seismic modelling relies on the use of complex velocities. Since the pioneering work of G. Pratt and co-workers, several promising applications of 2D acoustic frequency-domain FWI have been published, where only the dominant P-wave velocity (V_P) parameter is reconstructed (e.g., Ravaut et al., 2004; Gao et al., 2006; Jaiswal et al., 2009). Today, two challenges for FWI are reconstruction of multiple classes of parameters with contrasted signatures in the data, such as the S wave velocity (V_S), density, attenuation and anisotropy aside the P-wave velocity and the extension to 3D where the computational burden of multiple-source modelling is a key issue. The aim of the SEISCOPE project has been to tackle these two challenges since 2006. The main results obtained so far are reviewed in the following.

Multiparameter FWI

Most of recent applications of FWI have been performed in the acoustic approximation, where only the P-wave velocity is reconstructed. The acoustic approximation leads to inaccuracies in the amplitude modelling, the impact of which can be more or less significant depending on the medium properties (Barnes and Charara, 2009). The amplitude errors can be a dominant factor which prevent reliable FWI at high frequencies. Since 2006, we have developed a 2D massively parallel FWI code for imaging visco-elastic media where V_P, V_S, the density and the attenuation factors Q_P and Q_S can be reconstructed (Brossier, 2009). This code has been recently extended to vertically transversally isotropic (VTI) media by Gholami et al. (2010) while Operto et al. (2009) discussed the accuracy of the acoustic seismic modelling in TTI media. So far, we mainly focused on the joint reconstruction of V_P and V_S by FWI.

Preconditioned L-BFGS optimization

The inversion relies on a preconditioned L-BFGS quasi-Newton algorithm that allows one to compute without significant extra computational cost an approximation of $\mathbf{HG}$ where $\mathbf{H}$ is the inverse of the Hessian and $\mathbf{G}$ is the gradient of the misfit function (Nocedal, 1980). In the framework of multiparameter FWI, an approximate Hessian allows one to to scale the gradients associated with each parameter class before the estimation of the step length. The L-BFGS algorithm recursively provides an approximation of $\mathbf{HG}$ from the gradients and the models of previous iterations by performing inner products of vectors. If different classes of parameters with different order of size are involved in the inversion, estimation of these inner products can be biased towards the dominant parameters. A simple and efficient way to scale properly the L-BFGS optimization is to make the different parameter classes adimensional before estimation of $\mathbf{HG}$ (Brossier, 2009). The gradient and the Hessian of the misfit function for adimensional parameters are: $\frac{\partial \mathcal{C}(\mathbf{m})}{\partial \bar{\mathbf{m}}} = m_0 \frac{\partial \mathcal{C}(\mathbf{m})}{\partial \mathbf{m}} \quad \frac{\partial^2 \mathcal{C}(\mathbf{m})}{\partial \bar{\mathbf{m}}^2} = m_0^2 \frac{\partial^2 \mathcal{C}(\mathbf{m})}{\partial \mathbf{m}^2}$, equation 1, where $\bar{\mathbf{m}} =$

$\left[\bar{\mathbf{m}}^{(1)}/m_0^{(1)}\bar{\mathbf{m}}^{(2)}/m_0^{(2)}...\bar{\mathbf{m}}^{(n)}/m_0^{(n)}\right]$ is the adimensional model obtained by scaling the n parameter classes of the original model by the corresponding mean value m_0. Equation 1 shows that the gradient and the preconditioner of the Hessian provided by the diagonal approximation of the Hessian must be scaled by m_0 and m_0^2 before performing the L-BFGS optimization while the estimation of $\mathbf{HG}$ returned by the L-BFGS must be unscaled by multiplication by m_0.

Elastic FWI: onshore application
We first applied elastic FWI to a realistic synthetic onshore case study, that corresponds to a dip section of the 3D acoustic EAGE/SEG overthrust model (Brossier et al., 2009c). An elastic model was built by using a constant Poisson ratio of 0.24. The acquisition design is a wide-aperture acquisition with a maximum offset of 20 km. The challenge for this application is the joint inversion of body waves and surface waves in presence of a heterogeneous weathered layer in the near surface. We have concluded that successfull reconstruction of both V_P and V_S models requires to introduce two levels of data preconditioning in the inversion implemented into two nested loops in the algorithm. The first level of data preconditioning proceeds as usually over increasing frequencies while the second one proceeds over decreasing time dampings to introduce progressively in the inversion shorter-aperture and more complex late-arriving phases such as surface waves and PS-converted waves. Successive inversions of overlapping frequency groups rather than successive inversions of single frequencies is another key factor which contributed to significantly improve the S/N ratio of the reconstructed models. Two important conclusions of this application are that V_P and V_S must be reconstructed simultaneously because of the strong footprint of surface waves in the seismic wavefield and that V_S can require a more accurate starting model than V_P because of the higher-resolution power of the low shear-wave velocities.

Elastic FWI: offshore application
Second, we imaged the offshore Valhall model by elastic FWI from a 4-C OBC survey with a maximum offset of 16 km (Brossier et al., 2009a). The Valhall model is mainly characterized by a soft sea bed which prevents a large amount of PS conversions and the presence of low-velocity gas layers above the reservoir level. A first conclusion is that acoustic FWI of the hydrophone component of the elastic data provides a reliable V_P model. This V_P model can be used as a starting model to perform the joint reconstruction of V_P and V_S from the geophone components. While the onshore case study required the joint reconstruction of V_P and V_S, offshore applications where V_S can have a minor footprint in the seismic wavefield, can require a hierarchical approach where the dominant V_P parameter is first reconstructed followed by the joint reconstruction of V_P and V_S.

Sensitivity of FWI to noise
The least-squares L_2 norm remains the most popular optimization criterion, although its sensitivity to large errors in the data. We (re-)investigated alternative functionals for FWI such as the least-absolute-value L_1 norm, the Huber norm and an hybrid L_1-L_2 criterion (Brossier et al., 2009b). Our conclusion is that the L_1 norm provides the most robust and easy-to-tune criterion. We applied the L_1 norm in the framework of efficient frequency-domain FWI (i.e., waveform inversion applied to strongly decimated data) to the overthrust and Valhall case studies where random white noise is added to the data. In the overthrust experiment, the L_1 norm outperforms L_2 norm when the same number of frequencies is used in the inversion. If the number of inverted frequencies is increased when the L_2 norm is used, the L_2 norm can provide results as good as the L_1 norm at the expense of the computational cost. In the Valhall case, the L_2-norm inversion failed to successfully reconstruct the V_S model for data with white noise, while the L_1 norm provided satisfying V_P and V_S models even in presence of outliers.

Three dimensional FWI

Three-dimensional acoustic FWI is today feasible and promising applications to real data case studies at low frequencies (< 7 Hz) were recently published (Sirgue et al., 2009; Plessix, 2009). Because the inversion has been limited to rather low frequencies, FWI is viewed as a tool to build high-resolution velocity models than can be used as background model for migration.

Seismic modelling
The computational cost of 3D FWI is mainly controlled by the computational efficiency of the seismic modelling. Several modelling in the time domain and in the frequency domain have been proposed to perform FWI in the frequency domain (Virieux et al. (2009) for a review). The frequency-domain approach based on direct solver, widely used in 2D, has been extended to 3D by Operto et al. (2007). Although the direct-solver approach is prohibitively memory and time expensive and poorly scalable for large-scale problems involving several tens of millions of unknowns, it allowed us to tackle efficiently problems of few millions of unknowns involving a large number of sources (Ben-Hadj-Ali et al., 2008). These problems are representative of petroleum targets at low frequencies ($<$ 7 Hz). To tackle larger-scale problems, we developed a domain decomposition algorithm for 3D acoustic wave modelling based on the substructuring Schur complement approach and hybrid direct-iterative solver (Sourbier et al., 2008). The hybrid strategy is more scalable and less memory demanding that the direct approach but the source-dependent task (i.e., iterative solve of the Schur complement system) is more computationally demanding that of the direct-solver approach (i.e., substitution phase). This prompt us to assess source encoding to mitigate the computational cost of multi-source modelling in frequency-domain FWI.

Speeding up FWI by source encoding
Taking advantage of the linear relationship between the seismic wavefield and the source, sources of a seismic survey can be stacked before seismic modelling (Capdeville et al., 2005). The resulting seismic wavefield is the sum of the wavefields associated with each individual sources and the gradient of the misfit function built from the stacked wavefield is the gradient of the misfit function built from individual wavefields (i.e., the desired gradient) plus crosstalk terms resulting from the correlation between an incident wavefield associated with one source and the adjoint wavefield associated with another source. Random phase shifts can be applied to each individual sources before stack to mitigate the crosstalk noise while leaving unchanged the desired gradient (Romero et al., 2000; Krebs et al., 2009; Herrmann et al., 2009). We have assessed the source encoding technic in the framework of efficient frequency-domain FWI (Ben-Hadj-Ali et al., 2009), while the source encoding technic was applied to time-domain FWI by Krebs et al. (2009). We have highlighted the sensitivity of the method to noise when only few discrete frequencies are inverted sequentially and when all the sources are stacked in one single super-shot. An acceptable S/N ratio is obtained at the expense of the computational cost when frequency groups with sufficiently-fine frequency intervals are successively inverted rather than single frequencies. This can be an additional argument in favor of 3D time-domain modelling for frequency-domain FWI where an arbitrary large number of frequencies can be extracted at a minimum extra cost by discrete Fourier transform (Sirgue et al., 2008). Our conclusion is that the most efficient modelling strategy for 3D frequency-domain acoustic FWI might be the direct-solver approach for the inversion of the low frequencies ($<$ 7 Hz) combined with time-domain modelling and source encoding to process higher frequencies. For 3D elastic wave modelling, the direct-solver approach is too computationally expensive and memory demanding for the moment. In the prospect of 3D elastic FWI, we have developed a low-order hp-adaptive finite-element discontinuous Galerkin method in the time-space domain on unstructured tetrahedric meshes, which allows us to locally refine the mesh and tackle complex topographies and the fluid-solid interface in onshore and offshore environments, respectively (Etienne et al., 2009).

Conclusion and perspectives

FWI has become a mature tool to build high-resolution velocity models in the acoustic approximation for prestack-depth migration. The next challenges are the reconstruction of multiple classes of parameters such as V_S for detection of Poisson ratio anomalies in presence of gas and fluids, density and attenuation for a more accurate processing of amplitudes, anisotropy for processing of wide aperture/wide-azimuth data. Building reliable starting models for FWI remains an open question. Stereotomograpy based on the joint use of refraction and reflection traveltimes is one of our field of investigation.

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