

Lithospheric imaging from teleseismic data by frequency-domain elastic full-waveform tomography

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Abstract We shall present a 2D elastic frequency-domain full-waveform tomography method suitable for lithospheric imaging from teleseismic data. In the teleseismic configuration, the source is a plane wave impinging the base of the lithospheric target located below the receiver network. The plane-wave source is implemented in the frequency-domain forward problem using a scattered-field formulation. The wave modeling is performed with a finite-element discontinuous Galerkin method on unstructured triangular meshes. The inverse problem is solved in the frequency-domain using a quasi-Newton L-BFGS optimization and the adjoint-state method. Preliminary applications in the framework of the acoustic approximation were presented to highlight the resolution improvements provided by the inversion of topside reflections after the first reflection at the free surface. These shorter-aperture converted phases increase dramatically the high-wavenumber coverage in the model space which would have been rather poor otherwise. We assess in a realistic teleseismic setting for a 0.2-2 Hz source bandwidth the frequency sampling required for avoiding wraparound of lithospheric reflectors, which result from the narrow aperture illumination provided by plane wave sources when temporal frequencies are not sufficiently finely sampled. Before considering application to real data, obliquity of plane wave sources with respect to the imaged section must be addressed either by implementation of the 2.5D wave equation or by applying empirical corrections to velocities.

Introduction

With the development of dense multi-component broadband seismological networks, teleseismic data set become amenable to multichannel processing such as migration and full waveform tomography (FWT) for building high-resolution P- and S-wave velocity models of lithospheric targets. We have adapted a 2D frequency-domain FWT method, initially designed for controlled-source seismology, to teleseismic configuration where the seismic sources are a plane waves which impinge the base of the lithosphere with an arbitrary incidence angle. Our goal is to tend towards the full exploitation of the information contained in the data to improve the imaging resolution. A first attempt towards this goal was proposed by Bostock et al. (2001); Shragge et al. (2001) who adapted ray+Born migration/inversion to scattered teleseismic data contained in the coda of the P-wave (Bostock et al., 2001; Shragge et al., 2001). With this approach, Rondenay et al. (2001) successfully imaged the subducting Juan de Fuca plate down to 40-km depth and the upper mantle of the oceanic crust down to 100-km depth in the Cascadia subduction zone. A key issue in their approach is to exploit the PP and PS reflections from the free surface to mimic a controlled-source reflection experiment and, therefore, improve the resolution of the imaging by migration of back-scattered events from the deep reflectors. Compared to ray-based migration technique, FWT does not rely on the single-scattering approximation in the forward problem and can model accurately wavefields that propagate over a broad range of incidence angles and scatters with large diffraction angle. This implies that FWT can invert both forward-scattered and backward-scattered data. Parameterization of model parameters in terms of absolute values rather than relative values is another advantage of FWT compared to migration.

In this study, we assume that the source is far enough for allowing the plane wave approximation to be considered as the input source when the seismic P wave penetrates the lithosphere beneath the receiver array (Figure 1). This plane wave has an incidence angle which is directly connected to the epicentral distance between the earthquake and the barycentric point of the array through standard global models such as the PREM model (Dziewonski and Anderson, 1981). The method was developed first in the acoustic approximation using a finite-difference frequency-domain modelling engine and was extended to the P-SV case using a finite-element discontinuous Galerkin method on unstructured triangular meshes. In this study, we present preliminary applications of acoustic FWT to highlight the importance of free-surface reflections to improve imaging resolution and the footprint of the limited aperture illumination provided by the teleseismic configuration.

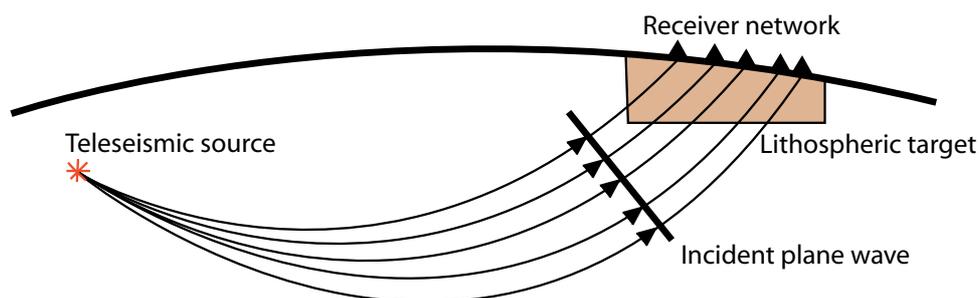


Figure 1 Sketch of a teleseismic experiment. The teleseismic wavefield can be processed as a plane-wave source that impinges the bottom of the lithospheric target located below the receiver array.

Forward problem

For P-SV wave modeling using plane wave sources, we use a hp -adaptive frequency-domain finite-element discontinuous Galerkin method on unstructured triangular mesh developed by Brossier et al. (2008); Brossier (2009). The first-order hyperbolic elastodynamic system is recast in pseudo-conservative

form as:

$$\begin{aligned}
 -i\omega\rho V_x &= s_x \frac{\partial(T_1 + T_2)}{\partial x} + s_z \frac{T_3}{\partial z} \\
 -i\omega\rho V_z &= s_x \frac{T_3}{\partial x} + s_z \frac{\partial(T_1 - T_2)}{\partial z} \\
 -i\omega \frac{T_1}{\lambda + \mu} &= s_x \frac{\partial V_x}{\partial x} + s_z \frac{\partial V_z}{\partial z} \\
 -i\omega \frac{T_2}{\mu} &= s_x \frac{\partial V_x}{\partial x} - s_z \frac{\partial V_z}{\partial z} \\
 -i\omega \frac{T_3}{\mu} &= s_x \frac{\partial V_z}{\partial x} - s_z \frac{\partial V_x}{\partial z},
 \end{aligned} \tag{1}$$

where V_x and V_z , are the components of the particle velocity vector, and T_1 , T_2 and T_3 are the components of the stress vector as $\vec{T}^t = (T_1, T_2, T_3) = ((\sigma_{xx} + \sigma_{zz})/2, (\sigma_{xx} - \sigma_{zz}), \sigma_{xz})$. s_x and s_z are functions describing the PML absorbing conditions (Berenger, 1994). The density is denoted by ρ , and the angular frequency by ω . The system, equation 2, can be recast in matrix form $\mathbf{Ax} = \mathbf{s}$, where \mathbf{A} is the complex-valued impedance matrix, \mathbf{x} is the velocity-stress wavefield vector and \mathbf{s} is the source vector. To implement plane wave source, we used a scattered-field formulation (Taflove and Hagness, 2000): the wavefield computed in an arbitrary heterogeneous medium is written as the sum of an incident wavefield and the scattered wavefield, noted \mathbf{x}_{inc} and \mathbf{x}_{sc} , respectively. Here, the incident wavefield is computed analytically in a homogeneous half space with a planar free surface. Within the lithospheric target (i.e., outside the area of source excitation), the incident and the scattered wavefields satisfy:

$$\begin{aligned}
 \mathbf{Ax} &= \mathbf{0} \\
 \mathbf{A}_0 \mathbf{x}_{inc} &= \mathbf{0},
 \end{aligned} \tag{2}$$

where A_0 denotes the impedance matrix computed for the homogeneous half space. Injecting the expression of the full wavefield, $\mathbf{x} = \mathbf{x}_{inc} + \mathbf{x}_{sc}$, in the first equation of the system 2 and taking the difference between the two equations of the system gives the system for the scattered wavefield:

$$\mathbf{Ax}_{sc} = -(\mathbf{A} - \mathbf{A}_0)\mathbf{x}_{inc}, \tag{3}$$

where the source in the right-hand side term is non zero only where the true medium differs from the homogeneous half space. Once the scattered wavefield has been computed, the full wavefield can be computed by summation of the incident wavefield with the scattered wavefield.

Inverse problem

In this study, we implement FWT in the frequency domain following the multiscale approach promoted by Pratt et al. (1998): increasing frequencies are successively inverted to inject progressively small wavenumbers in the model and, therefore, mitigate the non linearity of the inverse problem. The gradient of the misfit function is computed with the adjoint-state method (Plessix, 2006) and the optimization is performed with the L-BFGS quasi-Newton method (Nocedal and Wright, 1999) preconditioned by the diagonal terms of the Hessian. The definition of the initial model is a critical issue in FWT. The background reference model for the forward problem is presently disconnected to the starting model for FWT except at the bottom of both models where the same constant velocity is considered.

Resolution analysis of teleseismic FWT

In the framework of diffraction tomography, it can be shown that the wavenumber vector \vec{k} , locally imaged at a diffractor point in the medium, is related to the angular frequency ω and the diffraction angle (or aperture) θ by $\vec{k} = \frac{2\omega}{c} \cos\left(\frac{\theta}{2} \vec{n}\right)$, where c is the wavespeed and \vec{n} is a unit vector in the direction of the slowness vector formed by the sum of the slowness vectors associated with the source-diffractor and the receiver-diffractor ray paths. This relation highlights the redundant control of frequency and

aperture on wavenumber coverage. Higher wavenumbers are injected in the FWT model as the inversion proceeds towards higher frequencies and smaller apertures. Teleseismic experiments are characterized by a narrow illumination of apertures because of the limited number of plane wave sources related to the teleseismic earthquake distribution. Plane wave propagation from the bottom of the model to the receiver array on the free surface mainly leads to limited illumination pattern (i.e. forward-scattering wavepaths) which limits the ability of FWT to resolve the short wavelengths of the lithospheric model. The narrow aperture bandwidth requires the use of finely-sampled frequencies in FWT to prevent spatial aliasing in the reconstructed FWT models. This feature contrasts with the strategy of selecting a limited number of frequencies as applied in the framework of surface-surface FWT (Sirgue and Pratt, 2004) where the wide-aperture configuration is found. In order to overcome the limited resolution power of coarse and narrow illumination sampling, it is crucial to consider all reflections and conversions from the lithospheric reflectors, generated after a first reflection from the free surface.

Numerical examples in the acoustic approximation

We first consider a dip section of the 3D EAGE/SEG overthrust model at the oil-exploration scale to highlight how reflections from the free surface help the improvement of the resolution of the velocity model (Figure 2a). The data were computed in the acoustic approximation and only the P-wave velocity is reconstructed. The target is illuminated by thirteen plane waves with incident angles ranging from -30° to 30° . Seven frequencies between 4 and 15 Hz are successively inverted. An initial model is built by Gaussian smoothing of the true model with correlation lengths of 500 m (Figure 2b). Comparison of the final FWT models obtained from data computed with and without free surface effects shows the resolution improvement provided by short-aperture topside reflections from the overthrust reflectors (Figure 2(c-d)) as well as the more accurate estimation of the velocities (Figure 2(e-h)).

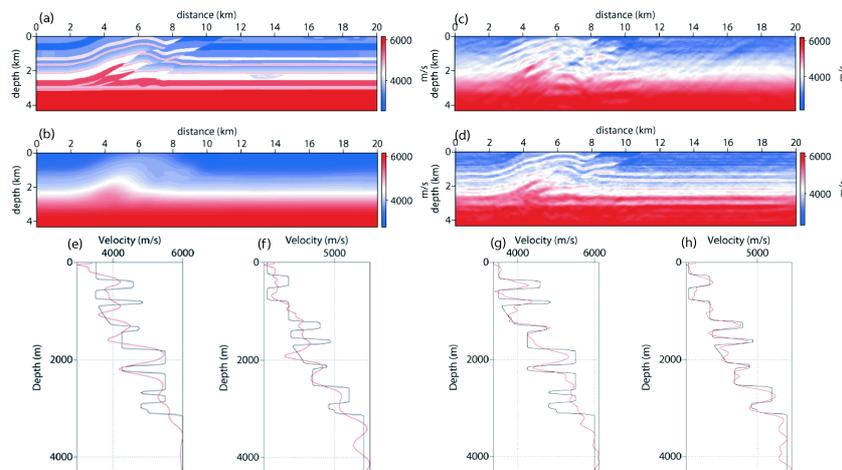


Figure 2 Results of the overthrust experiment. (a) True model. (b) Starting model for FWT. (c-d) FWT models inferred from data computed without (c) and with (d) free surface condition. (e-f) Comparison between velocity logs extracted from the true model (black line) and the FWT model of (c) (red line) at distances 4.35 km (e) and 10 km (f). (g-h) Same as (e-f) for the FWT model of (d).

We now consider a lithospheric target, that represents a suture model, composed of crustal material, upper mantle and a high-velocity relict crust (Shragge et al., 2001) (Figure 3a). We want to assess the frequency sampling required by the teleseismic acquisition configuration to avoid spatial aliasing of the imaged structures (Plessix and Mulder, 2004). The starting model used for acoustic FWT is shown in Figure 3. We used 9 plane waves with incidence angles between -30 to 30 degrees. We inverted respectively 7 and 24 frequencies between 0.2 and 2 Hz. The final FWI models for the P-wave velocity are shown in figure 3(c-d). Wraparound of the Moho reflector is clearly visible in figure 3(c), in contrast to figure 3(d) where a finer frequency interval was used. Of note, the wraparound problem is even more severe in the case of the S-wave velocity parameter because of shorter propagated wavelengths.

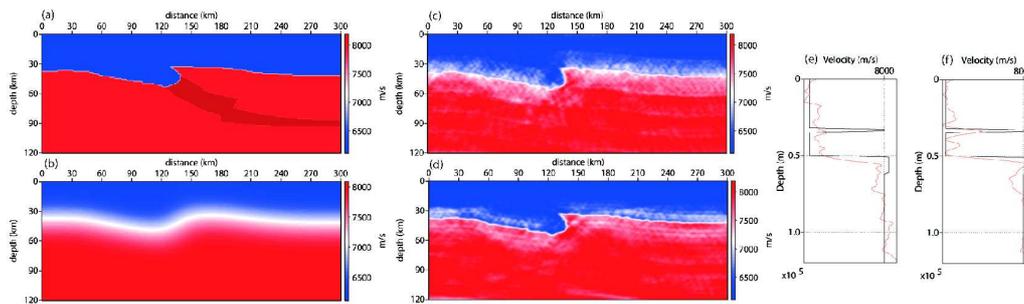


Figure 3 Imaging of the suture model. (a) True model. (b) Starting model for FWT. (c) Final FWT model when 7 frequencies between 0.2 and 2 Hz are inverted. (d) Same as (c) when 24 frequencies between 0.2 and 2 Hz are inverted. (e) Comparison between velocity profiles extracted from the true model (black line) and the FWT model (red line) of (c). (f) Same as (e) for the FWT model of (d).

Conclusions

We have presented a methodology for 2D multiscale elastic frequency-domain FWT of teleseismic data for lithospheric imaging. Preliminary applications in the framework of the acoustic approximation were presented to highlight the resolution improvements provided by the inversion of topside reflections after the first reflection at the free surface. These converted phases increase dramatically the illumination of the wavenumber spectrum which would have been rather poorly sampled otherwise. We assess in a realistic teleseismic setting the required frequency sampling for avoiding spatial aliasing of lithospheric reflectors. Perspectives of this work is the finalization of the visco-elastic FWT using a frequency-domain discontinuous Galerkin forward problem on unstructured triangular meshes and the definition of optimal strategy (empirical corrections of velocity, 2.5D wave equation) to account for the obliquity of the plane wave sources with respect to imaged plane before considering application to real data.

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References

- Berenger, J.P. [1994] A perfectly matched layer for absorption of electromagnetic waves. *Journal of Computational Physics*, **114**, 185–200.
- Bostock, M.G., Rondenay, S. and Shragge, J. [2001] Multiparameter 2D inversion of scattered teleseismic body waves 1. theory for oblique incidence. *Journal of Geophysical Research*, **106**(12), 30771–30782.
- Brossier, R. [2009] *Imagerie sismique à deux dimensions des milieux visco-élastiques par inversion des formes d'onde: développements méthodologiques et applications*. Ph.D. thesis, Université de Nice-Sophia-Antipolis.
- Brossier, R., Virieux, J. and Operto, S. [2008] Parsimonious finite-volume frequency-domain method for 2-D P-SV-wave modelling. *Geophysical Journal International*, **175**(2), 541–559.
- Dziewonski, A.M. and Anderson, D.L. [1981] Preliminary reference Earth model. *Phys. Earth Planet. Interiors*, **25**, 297–356.
- Nocedal, J. and Wright, S.J. [1999] *Numerical Optimization*. New York, US : Springer.
- Plessix, R.E. [2006] A review of the adjoint-state method for computing the gradient of a functional with geophysical applications. *Geophysical Journal International*, **167**(2), 495–503.
- Plessix, R.E. and Mulder, W.A. [2004] Frequency domain finite difference amplitude preserving migration. *Geophysical Journal International*, **157**, 975–987.
- Pratt, R.G., Shin, C. and Hicks, G.J. [1998] Gauss-Newton and full Newton methods in frequency-space seismic waveform inversion. *Geophysical Journal International*, **133**, 341–362.
- Rondenay, S., Bostock, M.G. and Shragge, J. [2001] Multiparameter 2D inversion of scattered teleseismic body waves 3. Application to the Cascadia 1993 data set. *Journal of Geophysical Research*, **106**(12), 30795–30807.
- Shragge, J., Bostock, M.G. and Rondenay, S. [2001] Multiparameter 2D inversion of scattered teleseismic body waves 2. Numerical examples. *Journal of Geophysical Research*, **106**(12), 30783–30793.
- Sirgue, L. and Pratt, R.G. [2004] Efficient waveform inversion and imaging : a strategy for selecting temporal frequencies. *Geophysics*, **69**(1), 231–248.
- Taflove, A. and Hagness, C. [2000] *Computational electrodynamics: the finite-difference time-domain method*. Artech House, London, United Kingdom.