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A Robust Absorbing Layer Method for Seismic Wave Simulation in Anisotropic Media

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SUMMARY

Seismic wave modeling requires using adapted boundary conditions to simulate infinite or semi-infinite media. Because of its efficiency, the Perfectly Matched Layers (PML) method has rapidly become the standard for acoustic and elastic propagation. However, PML are not adapted to anisotropic media for which the method becomes amplifying. Alternative methods have to be designed. In this study, we present the SMART layer method, which relies on a diagonal decomposition of the hyperbolic operator. The method is not perfectly matched, therefore less efficient than the PML method, however it is proved to remain dissipative, even for anisotropic media. We apply the method to the acoustic TTI equations. We present numerical results on a homogeneous test case and on the BP 2007 model, which includes a space dependent tilt angle. We compare the SMART and the PML methods. The results emphasize the robustness of the SMART method: no wave amplification is observed. In addition, the accuracy of the PML can be reached at the expense of an increase of the SMART layer width. The additional computational cost is compensated by the simple form of the SMART layer: only a zero-order term is added to the equations and no additional variables are required

Introduction

Modern seismic imaging methods such as Full Waveform Inversion (FWI) or Reverse Time Migration (RTM) rely on the ability to accurately model the propagation of waves within the subsurface. At the exploration scale, the subsurface is assimilated to a semi-infinite media: a free surface condition on top delineates the interface between the ground (or sea) and the air; the Earth is considered to extend infinitely in depth and lateral directions. Numerical simulation of wave propagation thus requires to design efficient absorbing boundary conditions to avoid spurious reflections at the boundaries of the computational domain.

Because of its efficiency and ease of implementation, the Perfectly Matched Layers (PML), introduced by Bérenger (1994), has rapidly became the standard. Originally designed for electromagnetism, the method has been successfully extended to seismic wave propagation in acoustic and elastic isotropic media (Collino and Tsogka, 2001). The interest domain is surrounded by an absorbing layer in which the waves are damped. The method is perfectly matched: the theoretical reflection coefficient at the interface between the interest domain and the layer is equal to zero for all incidence angles.

The extension of PML to seismic wave propagation in anisotropic media is however not satisfactory. When applied to these equations, PML become amplifying: instead of decaying, the amplitude of outgoing waves grows exponentially. This behavior has first been emphasized by Becache et al. (2003) in the context of 2D elastic Vertically Transverse Isotropic (VTI) media. More general results through high frequency asymptotic analysis have later been proposed by Halpern et al. (2011): the amplification of the waves is due to the change of the wave propagation operator induced by the PML method. Acoustic and elastic isotropic wave propagation operators are particular cases for which this amplification does not occur.

Nonetheless, accurately accounting for anisotropy in wave propagation modeling is crucial. Anisotropy is naturally described in linear elasticity through the definition of the stress-strain tensor. Acoustic anisotropic models have also been derived, since the work of Alkhalifah (2000), to focus on compression P-waves and alleviate the computational burden associated with elastic wave propagation modeling. The PML method exhibits amplifying behavior both for elastic and acoustic anisotropic wave propagation models.

In this study, we investigate the use of the SMART layer (Halpern et al., 2011) to efficiently simulate wave propagation in anisotropic media. The SMART layer is not perfectly matched, it is thus less efficient than the PML method in terms of spurious reflections. However, the main interesting property of the SMART layer is to guarantee no amplification even for anisotropic wave propagation models.

Method

We present an overview of the SMART layer method based on the 1D acoustic wave equation. A detailed presentation of the method can be found in Métivier et al. (2014). Consider the velocity-stress system

$$\partial_t u + A\partial_z u = s, \tag{1}$$

where

$$u = [u_z \;\; p]^T, \;\; s = [0 \;\; s_p]^T, \;\; A = -\begin{pmatrix} 0 & \frac{1}{\rho} \\ \rho c^2 & 0 \end{pmatrix}, \tag{2}$$

and $u_z(z,t)$ is the vertical velocity displacement, $p(z,t)$ is the stress, $c(z)$ is the wave velocity and $\rho(z)$ is the density. The matrix A is diagonalizable and its eigenvalues are c and $-c$. We denote u^+ and u^- the corresponding eigenvectors, and P the change of coordinates matrix such that $P^T = [u^+ \;\; u^-]$. We define v as Pu, solution of

$$\begin{array}{l} \partial_t v_1 = cv_1 + (Ps)_1 \\ \partial_t v_2 = -cv_2 + (Ps)_2. \end{array} \tag{3}$$

The system (3) consists in the superposition of the propagation of two plane waves: v_1 propagates downward, while v_2 propagates upward. The absorption of down-going and up-going waves is simply performed by the introduction of absorption terms $\sigma^+(z)v_1$ and $\sigma(z)^- v_2$ in the first and second equation respectively of the system (3). The function $\sigma^+(z)$ and $\sigma^-(z)$ are zero in the interest domain and grow smoothly in the bottom (respectively the top) absorbing layers. Defining the matrices

$$B^+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \; B^- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4}$$

the introduction of the absorbing layer thus amounts to the introduction of a zero-order term in the initial equation (1)

$$\partial_t u + A\partial_z u + \sigma(z)^+ P^T B^+ P u + \sigma^-(z) P^T B^- P u = s. \tag{5}$$

Extension to the multi-dimensional case is straightforward. Consider the 3D hyperbolic system

$$\partial_t u + \sum_{j=1}^{3} A_j \partial_j u = s. \tag{6}$$

We define P_j the change of coordinate matrices associated with the diagonalization of the matrices A_j, and $B_j^{\pm}$ the matrices associated with the selection of the positive or the negative eigenvalue of the matrix A_j. The SMART layer strategy amounts to the definition of the system

$$\partial_t u + \sum_{j=1}^{3} A_j \partial_j u + \sigma(x_j)^+ P_j^T B_j^+ P_j u + \sigma^-(x_j) P_j^T B_j^- P_j u = s. \tag{7}$$

This yields absorbing layers designed to absorb wave propagating in a direction normal to the interface between the interest domain and the layer. The reflection coefficient is non-zero as soon as the incidence angle is non-zero. This is the main drawback of the method. However, the stability of the method is ensured as soon as the system of equation (6) is symmetrizable, *i.e.* there exists a symmetric positive definite matrix S such that the matrices SA_j are symmetric. Elastic and acoustic VTI/TTI equations satisfy this assumption (Métivier et al., 2014). In this case, one can show through energy estimates that the zero-order terms $P_j^T B^{\pm} P_j u$ are dissipative and do not generate amplification, contrary to the PML.

Numerical results for general acoustic TTI equations

We consider the acoustic TTI equations derived from the linear elasticity equations (Duveneck and Bakker, 2011).

$$\left\{ \begin{array}{lcl} \partial_t u_x & = & \frac{1}{\rho}\left[\partial_x\left(\cos^2\theta\sigma_{xx} + \sin^2\theta\sigma_{zz}\right) + \partial_z\left(\sin\theta\cos\theta\left(\sigma_{zz} - \sigma_{xx}\right)\right)\right] \\ \partial_t u_z & = & \frac{1}{\rho}\left[\partial_z\left(\sin^2\theta\sigma_{xx} + \cos^2\theta\sigma_{zz}\right) + \partial_x\left(\sin\theta\cos\theta\left(\sigma_{zz} - \sigma_{xx}\right)\right)\right] \\ \partial_t \sigma_{xx} & = & \rho v_P^2 (1+2\varepsilon)\left[\cos^2\theta\partial_x u_x - \sin\theta\cos\theta(\partial_x u_z + \partial_z u_x) + \sin^2\theta\partial_z u_z\right] \\ & & +\rho v_P^2\sqrt{1+2\delta}\left[\sin^2\theta\partial_x u_x + \sin\theta\cos\theta(\partial_x u_z + \partial_z u_x) + \cos^2\theta\partial_z u_z\right] \\ \partial_t \sigma_{zz} & = & \rho v_P^2\sqrt{1+2\delta}\left[\cos^2\theta\partial_x u_x - \sin\theta\cos\theta(\partial_x u_z + \partial_z u_x) + \sin^2\theta\partial_z u_z\right] \\ & & +\rho v_P^2\left[\sin^2\theta\partial_x u_x + \sin\theta\cos\theta(\partial_x u_z + \partial_z u_x) + \cos^2\theta\partial_z u_z\right]. \end{array} \right. \tag{8}$$

The anisotropy symmetry axis is defined by the angle $\theta(x,z)$. The Thomsen anisotropy parameters are $\varepsilon(x,z)$ and $\delta(x,z)$, u_x, u_z are the velocity displacement in x and z direction, σ_{zz} is the stress aligned with the symmetry axis of anisotropy and σ_{xx} is the stress tangential to this direction. This system is discretized using a 4th order finite difference scheme based on the rotated staggered grid method of Saenger and Bohlen (2004) in space and a 2nd order leap-frog scheme in time.

We first compare the SMART layer method with the PML method in a homogeneous medium, such that $v_P = 2000$ m.$^{-1}$, $\rho = 1000$ kg.m^{-3}, $\varepsilon = 0.3$, $\delta = 0.1$, $\theta = 36^o$. We use a Ricker wavelet centered on 15 Hz as an explosive source located at the center of the domain ($x = 1000$ m and $z = 1000$ m). The comparison of snapshots of the wavefield at different time illustrates the amplification yielded by the PML method while the SMART layer properly decreases the energy of the solution. We compare the accuracy of the SMART layer method with the PML method. We modify the previous setting choosing $\varepsilon = \delta = 0.3$, (elliptical anisotropy) to ensure no amplification is caused by the PML. We move the source close to the surface located at $x = 1000$ m, $z = 100$ m and we locate receivers at the same depth, from $x = 0$ m to $x = 2000$ m to mimic a surface acquisition system. We impose a free-surface condition on

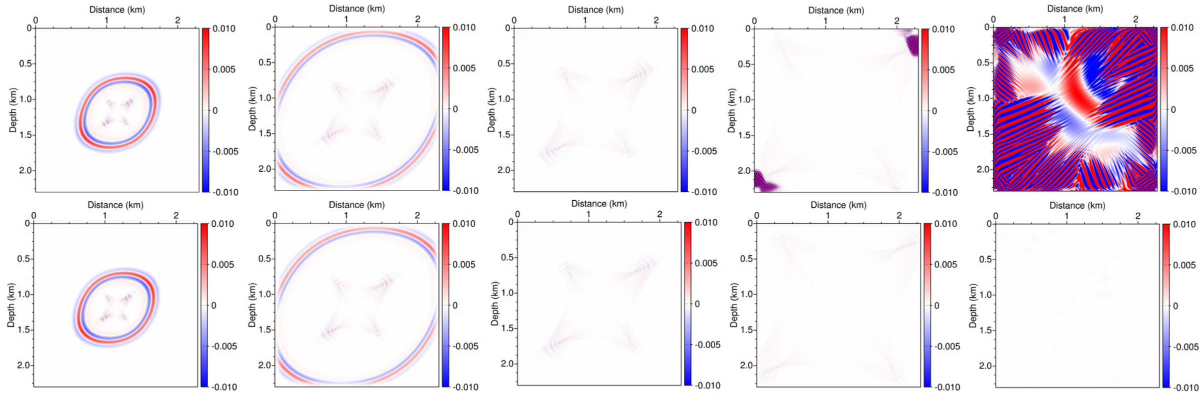

Figure 1 *Wavefield snapshot at 0.3 s, 0.6 s, 0.9 s, 1.5 s, 2.7 s for PML (top row), SMART layers (bottom row). The standard diamond shape of the S-wave artifacts due to the anellipticity can be observed at time $t = 0.3$ s. At time $t = 0.9$ s the P-waves have been efficiently absorbed both for PML and SMART strategies. The exponential growth of the PML solution is highlighted by wavefield snapshots for $t = 1.5$ s and $t = 2.7$ s.*

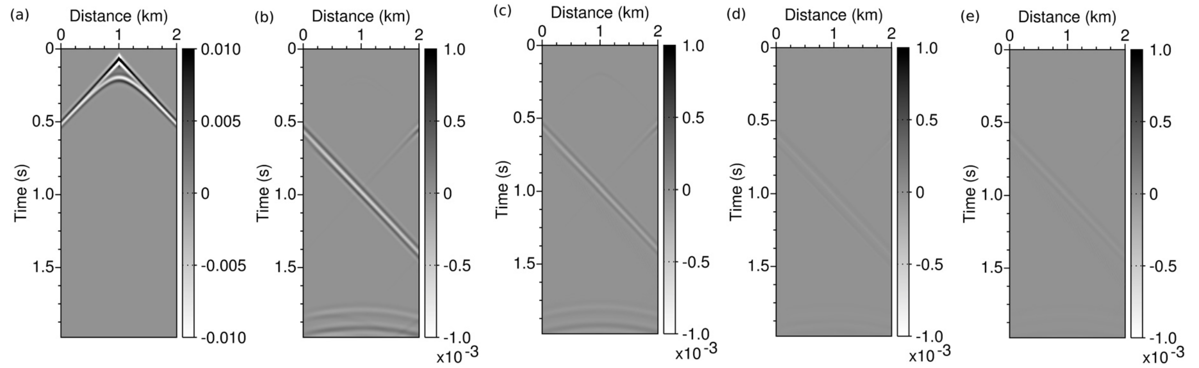

Figure 2 *Reference seismogram (a). Differential seismograms for SMART with 10 grid points (b), PML with 10 grid points (c), SMART with 15 grid points (d), PML with 15 grid points (e).*

top. We first compute a reference seismogram in a sufficiently large domain for a time window set to 2 s. We compare this seismogram with the ones obtained using SMART layers and PML with respectively 10 and 15 grid points in the layer (see fig. 2 where differential seismograms are presented). With 10 grid points in the layer, the PML yields a smaller amplitude spurious reflection. However for 15 grid points, the SMART and PML method yield similar results.

We finally compare the SMART and PML methods for the BP 2007 TTI benchmark model. The velocity v_P and anisotropy parameters ε, δ and θ are presented in figure 3. We use an explosive Ricker source centered on 4 Hz, at $x = 40$ km and $z = 100$ m. A free surface condition is implemented on top. We use a surface acquisition system with receivers at $z = 100$ m. The time window is set to 12 s. A reference solution is first computed in a large domain, and we compare seismograms and the wavefields at final time. The SMART layer method yields larger amplitude spurious reflections especially at non-normal incidence (differential seismograms, figure 4), however we can see that the amplification in the PML contaminates the interest domain even if it has not yet reached the receivers at $t = 12$ s (wavefields, figure 4).

Conclusion

The SMART layer method seems to be an alternative to the PML method for seismic wave modeling in anisotropic medium. Although less accurate than the PML method, especially for wave propagating at grazing angles, the SMART layer method is more robust as it ensures no amplification. Another interesting property is that the computation cost associated to the method is reduced compared to PML or C-PML as only one zero-order term is added and no splitting nor memory variables have to be used.

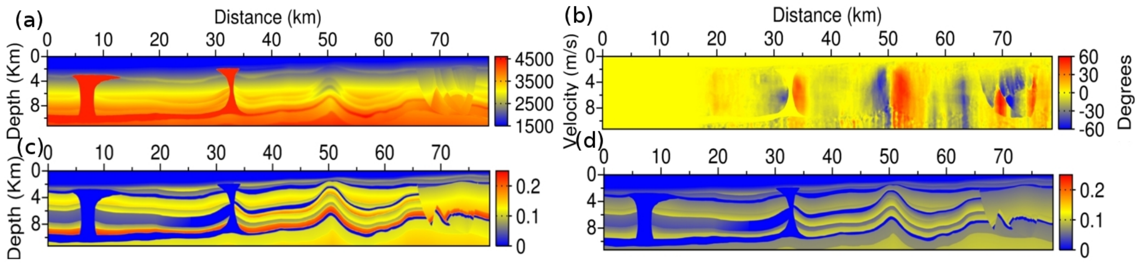

Figure 3 *BP 2007 benchmark model:* v_P*(a),* θ *(b),* ε *(c),* δ *(d)*

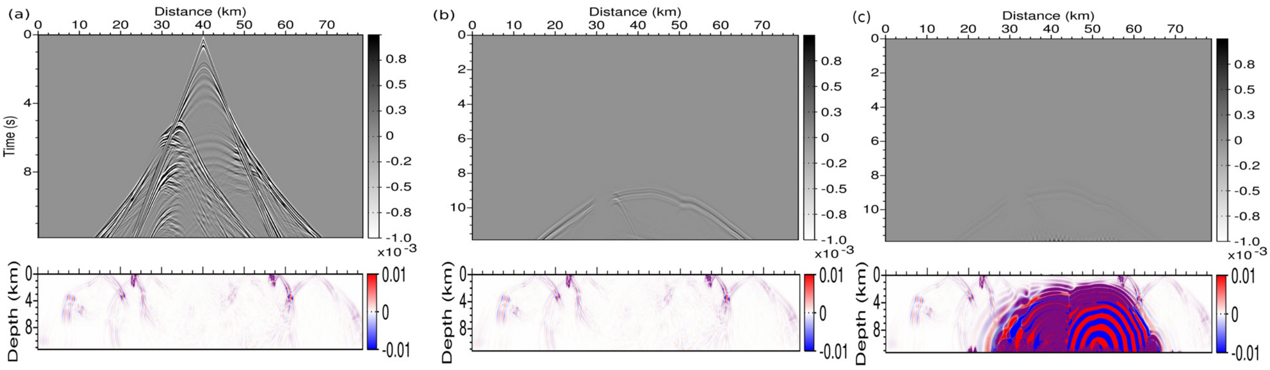

Figure 4 *Reference seismogram and reference wavefield in the interest domain at final time* $t = 12$ *s (a). Differential seismograms and wavefield in the interest domain at* $t = 12$ *s computed with SMART layers (b), PML (c). The exponential growth of the PML solution is visible in the wavefield snapshot although it has not yet reached the receiver zone and is not visible on the differential seismogram.*

Further studies will be performed to investigate if the accuracy of the method can be improved by extrapolation techniques (Halpern et al., 2011), or by combination with absorbing boundary conditions. At a longer term, tests will be performed to assess the interest of the method for seismic imaging within FWI or RTM methods.

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