C046

2D Frequency-domain Seismic Wave Modeling in VTI Media Based on a Hp-adaptive Discontinuous Galerkin Method

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SUMMARY

Seismic imaging in complex media by full waveform inversion requires efficient and accurate modelling tools to solve the full wave equation. In this study, we present a low order finite element discontinuous Galerkin (DG) method in the space-frequency domain, for 2D P-SV waves propagation in VTI media. DG method provides a high level of flexibility for simulation in complex media, thanks to the cell size and interpolation order adaptation that depends on local physical properties. The method is validated with a time-domain finite-difference method in an homogeneous strong anisotropy model and in a realistic heterogeneous model. Results show that VTI DG method allows to compute accurate solutions in complex media and is therefore suitable as a modelling engine for 2D elastic VTI full waveform inversion

Introduction

Seismic imaging of the subsurface is one of the main challenge for oil and gas reservoir characterisation. Frequency-domain full-waveform inversion (FWI) is a re-merging method to derive high-resolution quantitative models of the subsurface using the full wavefield computed in heterogeneous complex media (Pratt and Worthington, 1990).

During the last decade, most of the FWI applications to real data have been performed under the acoustic approximation to mitigate the computational burden of the full wave modelling and the non-linearity of the inverse problem. However, some geological setting should require to consider more complex media, which take into account elastic and anisotropic effects.

For anisotropic elastic FWI in complex onshore settings such as foothills, an efficient seismic modelling engine which can take into account topography of arbitrary shape must be developed.

Seismic wave propagation has been investigated with various numerical methods such as finite-difference (FD) or spectral-element (SE). The FD method is quite popular and have been intensively used for forward modelling as well as for seismic imaging but requires a high number of grid points per wavelength to keep numerical dispersion below acceptable levels especially at the free surface described by staircase approximation (Bohlen and Saenger, 2006). Low order Finite Element discontinuous Galerkin (DG) method have been recently developed (Brossier et al., 2008; Etienne et al., 2009) and was used as the forward problem for elastic isotropic FWI for onshore and offshore applications (Brossier et al., 2009).

In this study, we have developed a frequency-domain DG method for the 2D velocity-stress P-SV wave equation in vertically transverse isotropic (VTI) media. The method was validated against the classic FD time-domain method results and was implemented a the modelling engine of a 2D frequency-domain FWI algorithm for imaging VTI media.

Theory

To develop a frequency-domain DG method for wave modelling in 2D VTI media, we first need to consider the frequency-domain 1st order velocity-stress system in anisotropic media:

$$
\begin{aligned}
-i\omega V_x &= \frac{1}{\rho(\mathbf{x})}\left\{\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}\right\} + f_x \\
-i\omega V_z &= \frac{1}{\rho(\mathbf{x})}\left\{\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}\right\} + f_z \\
-i\omega \sigma_{xx} &= c_{11}\frac{\partial V_x}{\partial x} + c_{13}\frac{\partial V_z}{\partial z} - i\omega\sigma_{xx_0} \\
-i\omega \sigma_{zz} &= c_{13}\frac{\partial V_x}{\partial x} + c_{33}\frac{\partial V_z}{\partial z} - i\omega\sigma_{zz_0} \\
-i\omega \sigma_{xz} &= c_{44}\left\{\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right\} - i\omega\sigma_{xz_0},
\end{aligned}
\tag{1}
$$

where both particle velocities (V_x,V_z) and stresses (σ_{xx},σ_{zz},σ_{xz}) are unknown quantities. The c_{11},c_{13}, c_{33} and c_{44}, are stiffness coefficients for 2D VTI media, ρ is the density, ω is the angular frequency, and $i = \sqrt{-1}$ is the purely imaginary term. Source terms are either point forces (f_x, f_z) or applied stresses $(\sigma_{xx_0}, \sigma_{zz_0}, \sigma_{xz_0})$. Absorbing boundary conditions are implemented along the bottom, right and left edges of the computational domain with Perfectly-Matched Layers (Berenger, 1994) while a free surface boundary condition or an absorbing PML can be implemented on top of the model.

A change in variable is applied to the system of equations 1 in the same maner that Brossier et al. (2008) did for the isotropic case. We now consider the following vector with three components $(T_1, T_2, T_3) = ((\sigma_{xx} + \sigma_{zz})/2, (\sigma_{xx} - \sigma_{zz})/2, \sigma_{xz})$. Note that, when considering this variable change, T_1 represents the hydrostatic pressure measured in liquids. In the case of liquid/solid propagation, such as in marine seismic experiments, the T_1 variable provides direct access to the pressure measured by hydrophones, while T_2 and T_3 are zero in liquid, relative to the deviatoric part of the stress tensor.

The medium is discretised on unstructured triangular mesh. For each cell, the system of equations is multiplied by a test function, corresponding to a kth-order Lagrange polynomial. The test function is nonzero only in the polygonal cell that ensures the discontinuous property of the scheme. The system of

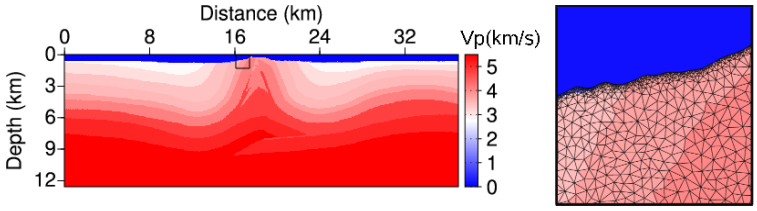

Figure 1 *The synthetic IFP/TOTAL Foothill P-wave velocity model representative of onshore targets with complex topographies and weathering zones, and a close-up of the mesh for DG P_2 simulations.*

equations is then integrated over each cell, which leads to the so-called weak formulation of the system. We assume that the physical properties of the medium are constant inside each cell, and we use centred numerical fluxes to exchange energy between cells (Remaki, 2000). In our algorithm, polynomials of order 0, 1 or 2 can be arbitrarily used in each cell leading to the so-called p adaptivity. Moreover, depending of the local physical properties, the cell size can be locally adapted in order to minimise the number of degree of freedom in the computational domain (h adaptivity). As an example, the Figure 1 shows a realistic onshore model discretised with an h-adaptive unstructured mesh that is suitable for the P_2 interpolation order. This mesh was designed for accurate modelling of the free-surface effects of the topography, and for the local adaptation of the cell size to the local wave speeds.
Discretisation of the system with the DG method leads to a sparse linear system $\mathbf{Au} = \mathbf{b}$ for each modelled frequency, where $\mathbf{A}$ is the so-called impedance matrix, $\mathbf{u}$ is the velocity-stress wavefield vector and $\mathbf{b}$ is the source vector. The system is solved with the massively parallel direct solver MUMPS (Amestoy et al., 2006) for efficient multi-source modelling required by FWI applications.

Numerical examples

We validate our VTI DG method against the established FD time-domain (FDTD) method for two cases. For both tests, our DG method is applied in the frequency-domain for all the frequencies of the source bandwidth. An inverse Fourier transform is then applied to frequency-domain solutions for comparison with time-domain FD results.

1) Strong anisotropy: Zinc model

A first validation test focuses on a strong anisotropic homogeneous model representative of the Zinc crystal (Komatitsch et al., 2000). The model dimensions are 2×2 km with a mesh size of 5 m for both FDTD and frequency-domain DG P_0. The source is in the middle of the grid. The source wavelet is a Ricker wavelet with a dominant frequency of 17 Hz. A horizontal receiver line is located above the source at a distance of 1 km. The physical properties of the Zinc crystal are outlined in table 1.
Figure 2 shows a frequency-domain monochromatic wavefield and comparison between DG and FD time-domain seismograms. A good agreement in phase and amplitude is obtained on the three components for the P-wave and the S-wave.

2) Anisotropic overthrust model

We then consider the anisotropic overthrust model (Figure 3). The dimensions of the overthrust model are 20 km x 4.38 km. PML are around the four edges of the model. Thomsen's parameters δ and ϵ range between -0.176602 and 0.06 and between 0 and 0.2, respectively. P-wave velocities range between 1650 m/s and 6000 m/s, while S-wave velocity is considered as constant and equal to 1300 m/s for comparison with acoustic modelling (Operto et al., 2009). The source is an explosion and the source wavelet is a Ricker wavelet with a dominant frequency of 4-Hz. The source, located at a horizontal distance of 10 km and at a depth of 0.4 km, is recorded by a line of 491 receivers at 300-m depth. The grid interval for both FDTD and DG P_0 methods are 10 m. Comparison between DG and FDTD seismograms for pressure and the vertical component of velocity is shown in Figure 4, where a good agreement between both solutions is obtained.

Medium	$V_p(m/s)$	$V_s(m/s)$	$\rho(\text{kg}/m^3)$	δ	ϵ
Zinc	2955.06	2361	7100	2.70968	0.830645

Table 1 *Physical properties of the Zinc homogeneous model*

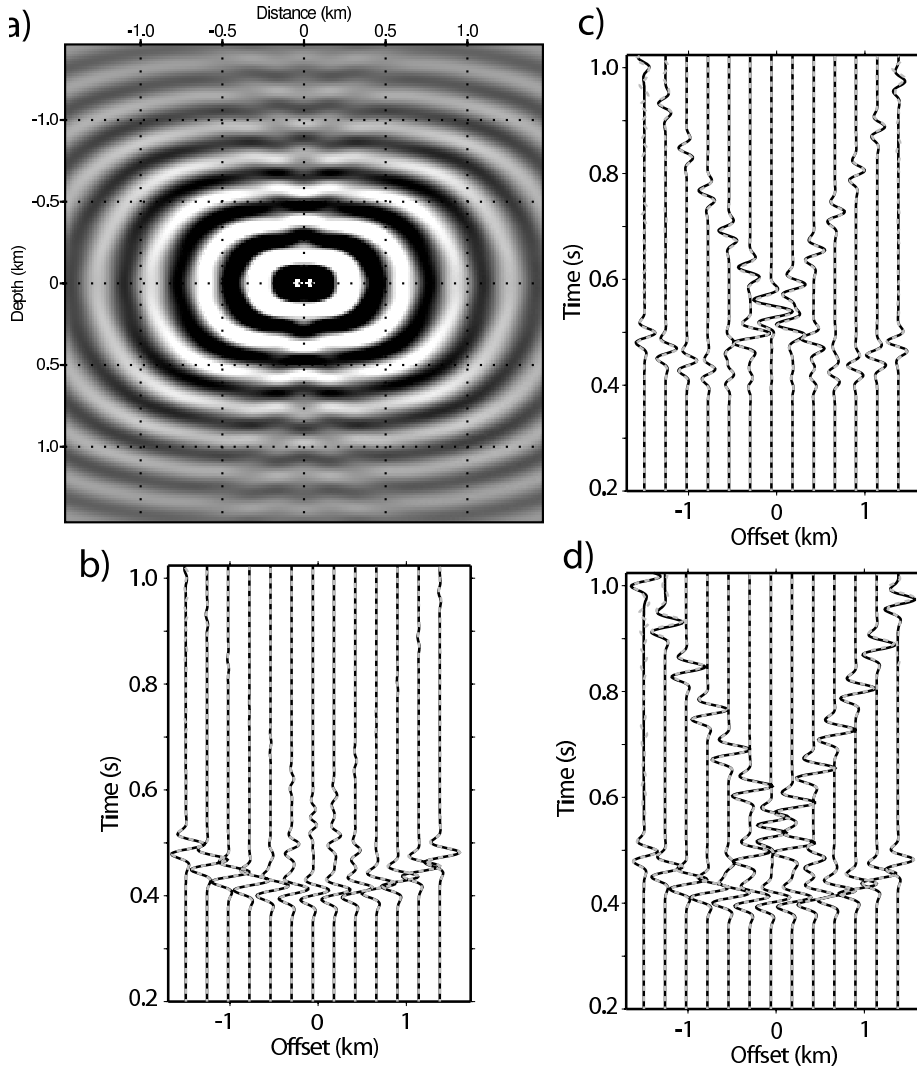

Figure 2 *Simulation in the Zinc model. a) Monochromatic wavefield for the T_1 component (pressure). Frequency is 21 Hz. b) c) and d) panels represent seismograms comparison for the pressure, the horizontal and the vertical components of velocity, respectively.*

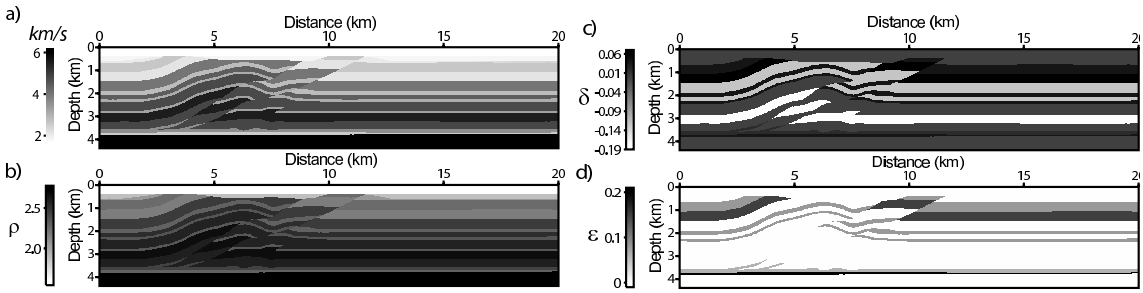

Figure 3 *Synthetic Overthrust model. a) V_P, b) density, (c-d) Thomsen's parameters: c) δ, d) ϵ*

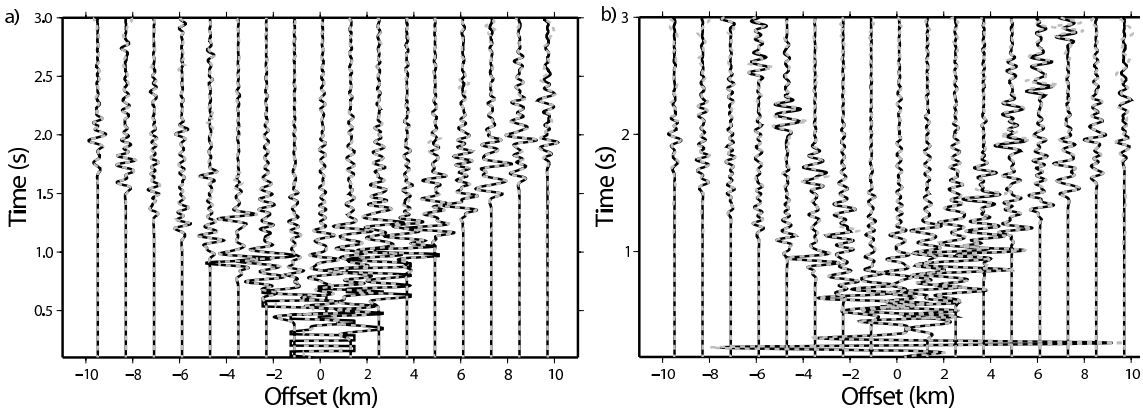

Figure 4 *Comparison between synthetic seismograms computed with the DGFD (dash gray) and the FDTD (solid black) methods. a) Pressure seismograms, b) Vertical particle velocities.*

Conclusion

A DG P_k method has been formulated in the space-frequency domain for 2D P-SV wave propagation in VTI media. The method has been validated with FDTD reference solutions in an homogeneous strong anisotropy model and in a realistic heterogeneous model. As for the isotropic case, the VTI DG formulation allows to compute accurate solutions in complex media and is suitable as a modelling engine for 2D elastic VTI FWI.

Future work will focus on extension the extension of the method to TTI media. FWI for imaging VTI media using the DG forward problem is under investigation as presented in a companion paper. Definition of a suitable set of parameter classes for reliable reconstruction of anisotropic parameters from wide-aperture acquisitions by FWI will be a central issue.

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