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2D Elastic Frequency-domain Full-waveform Inversion for Imaging Complex Onshore Structures

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SUMMARY

Quantitative imaging of the elastic properties of the subsurface is essential for reservoir characterization. We assess with a realistic synthetic example the potentialities of 2D elastic full-waveform inversion for imaging complex onshore structures. Full-waveform inversion of land data is challenging because of the increased non-linearity introduced by free surface effects such as surface waves. To mitigate theses non-linearities, different multiscale strategies are assessed. Numerical optimization relies on the L-BFGS Quasi-Newton method which outperforms more classic preconditioned conjugate-gradient one. Sequential inversions of increasing frequencies define the most natural level of hierarchy in the multiscale imaging. We show that this regularization is not enough for adequate convergence in the case of land data. A second level of hierarchy over aperture angles implemented with complex-valued frequencies is necessary and allows convergence of the inversion towards acceptable velocity models. Among the possible strategies for sampling frequencies in the inversion, successive inversion of slightly-overlapping frequency groups has proven to be the most reliable one when compared with more standard sequential inversion of single frequencies. This suggests that simultaneous inversion of multiple frequencies is critical when considering complex wave phenomena such as surface wave propagation.

Introduction

Quantitative seismic imaging of elastic parameters is one of the main challenge for oil and gas reservoir characterization. Frequency-domain full-waveform inversion (FWI) (Pratt and Worthington, 1990) allows to build accurate velocity models of complex structures from long offset acquisition geometries. Most of the applications of FWI to real data at different exploration scales are performed under the acoustic approximation (Ravaut et al., 2004; Operto et al., 2006) while only few papers tackle the elastic problem, found to be quite challenging (Gelis et al., 2007). First, the high numerical cost of the elastic forward problem, compared to the acoustic one, limits the size of tractable problems. Second, reconstruction of several classes of parameters partially coupled and the strong non-linearity associated with complex wave phenomena makes the convergence towards the global minimum of the objective function more difficult, in particular for land data for which both body waves and surface waves are involved in the inversion.
In this study, we present an application of frequency-domain elastic FWI to a dip section of synthetic Overthrust model in an onshore acquisition configuration. Forward problem is solved with frequency domain P_0 Discontinuous Galerkin method (Brossier et al., 2008). We investigate several multiscale strategies to mitigate the high non-linearity of the inverse problem. These strategies involve two nested levels of hierarchy over frequencies and aperture angles. Results show that simultaneous inversions of multiple frequencies and data preconditioning by time damping are critical to obtain reliable results when surface waves propagating in a heterogeneous near surface are present in the elastic wavefield.

Theory

Inverse problem

FWI (Pratt and Worthington, 1990) is an optimization problem which can be recast as a linearized least-squares problem which minimizes the misfit between the frequency-domain recorded data (d_{obs}) and modeled data (d_{calc}) at iteration k, defined by the objective function

$$\mathcal{C}^{(k)} = \frac{1}{2}(d_{obs} - d_{calc}^{(k)})^{\dagger}\mathbf{W}_d(d_{obs} - d_{calc}^{(k)}). \tag{1}$$

Superscript † indicates the adjoint (transpose conjugate) and $\mathbf{W}_d$ is the square of a diagonal weighting matrix applied to the misfit vector to scale the relative contributions of each of its components. The gradient $\mathcal{G}^{(k)}$ of the objective function is efficiently computed by adjoint-state technique which requires to solve only two forward problems per shot.
A second-order Taylor expansion of the objective function allows to find the perturbation model $\delta\mathbf{m}$ which minimizes the objective function assumed to be locally parabolic

$$\mathbf{B}^{(k)}\delta\mathbf{m} = -\mathcal{G}^{(\mathbf{k})}, \tag{2}$$

where $\mathbf{B}^{(k)}$ is the Hessian of the objective function. Steepest Descent or Conjugate Gradient methods preconditioned by the diagonal terms of an approximate Hessian are conventionally used in FWI (Pratt et al., 1998; Operto et al., 2006).
In this study, we use the Quasi-Newton L-BFGS method (Nocedal, 1980) which appears to be one of the most robust and efficient limited-memory Quasi-Newton algorithm to solve large-scale non-linear problems. L-BFGS method estimates inverse of Hessian matrix from a limited number of gradient difference and model difference vectors. Nocedal (1980) L-BFGS algorithm computes the perturbation vector from additions, differences and inner products of vectors without explictly building the inverse of the Hessian matrix. Time and memory requirements for L-BFGS are thus negligible in the FWI algorithm while taking benefit from the off-diagonal elements of Hessian. Accounting for the off-diagonal terms of the Hessian deconvolves the gradient from limited-bandwith effects and improves the convergence rate of the iterative process. We use the diagonal of Pseudo-Hessian (Shin et al., 2001) as initial guess for L-BFGS.

Data preconditioning

Non-linearity of FWI can be efficiently mitigated by selecting a subset of specific arrivals. Data preconditioning can be applied in the frequency domain by means of complex frequencies ($\omega + \iota\gamma$), which is equivalent to damp seismograms in the time domain (Brenders and Pratt, 2007). The Fourier transform of a signal $f(t)$ damped in time by $exp^{-\gamma(t-t_0)}$ is given by

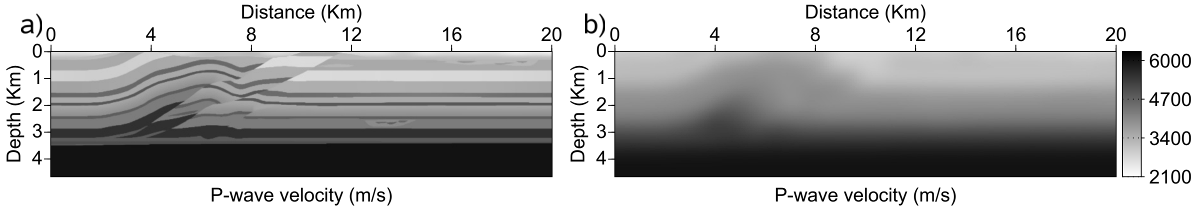

Figure 1: (a) True Overthrust P-wave velocity model and (b) related starting model for FWI.

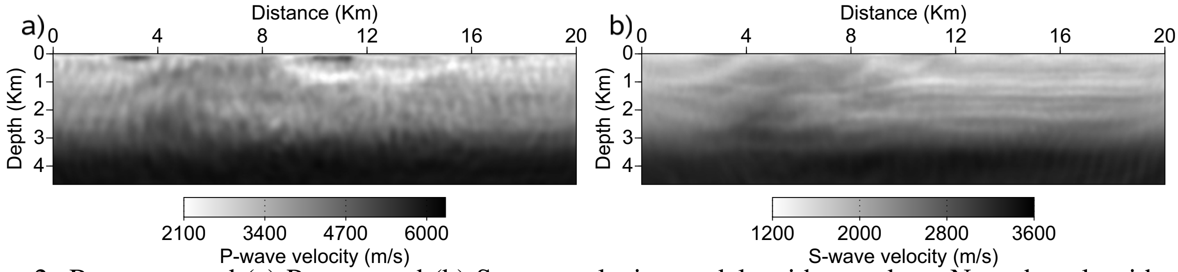

Figure 2: Reconstructed (a) P-wave and (b) S-wave velocity models with raw data. Note that algorithm fails in a local minimum.

$$F(\omega + \iota\gamma)exp^{\gamma t_0} = \int_{-\infty}^{+\infty} f(t)exp^{-\gamma(t-t_0)}exp^{-\iota\omega t}dt, \tag{3}$$

where the damping controlled by γ parameter, can be applied from an arbitrarily arrival time t_0 which usually corresponds to the first-arrival time. Time damping applied from the first-arrival time can be viewed as a heuristic way to select aperture angles in the data. Aperture selection can be exploited to implement a second level of hierarchy in the inversion in addition to frequency selection. Here, we use the complex-value frequencies to mitigate the impact of late-arriving phases such as surface waves and free surface reflexion during the early stages of the inversion. These waves add a significant level of non-linearity in the inversion.

Overthrust application

Acquisition geometry and inversion set-up

A dip section of SEG/EAGE overthrust model is considered (Figure 1-a). S-wave velocity model is built with a constant Poisson ratio (0.24) and a constant density is considered. 199 explosives sources are located 25 m bellow free surface and 198 geophones record horizontal and vertical particle velocity components at free surface. Starting models are smoothed version of true ones (Figure 1-b) and inversion is performed for 5 discrete frequencies: 1.7, 2.5, 3.5, 4.7 and 7.2 Hz. P- and S-wave velocities are the reconstructed parameters. 45 iterations are performed per frequency and per damping factor if convergence criterion is not reached before.

Raw data inversion

A first test consists of inverting raw data containing all arrivals. The five frequencies are inverted successively. Reconstructed models (Figure 2) clearly show that inversion converged towards a local minimum of the objective function far away from the global one, because of high surface wave residuals which dominate the backpropagated wavefields (Gelis et al., 2007).

Successive frequency inversions

To mitigate non-linearity associated with complex wave phenomena caused by free surface, complex frequencies are used to precondition data. Five damping factors (γ = 1.5, 1, 0.5, 0.1, 0.033) are used in cascade for each frequency to stabilize inversion. An updated model is obtained per inversion iteration of a complex-valued frequency. Frequencies are inverted sequentially defining a multiresolution reconstruction. Reconstructed models (Figure 3) illustrate the significant impact of the data preconditioning performed by hierarchical time damping. Most of the structures are well reconstructed with a reliable estimate of velocity amplitudes despite a low maximum frequency of 7.2 Hz. Note that both models are affected by inaccuracies in their shallowest part and S-wave velocity model exhibits spurious artefacts especially in the deep part.

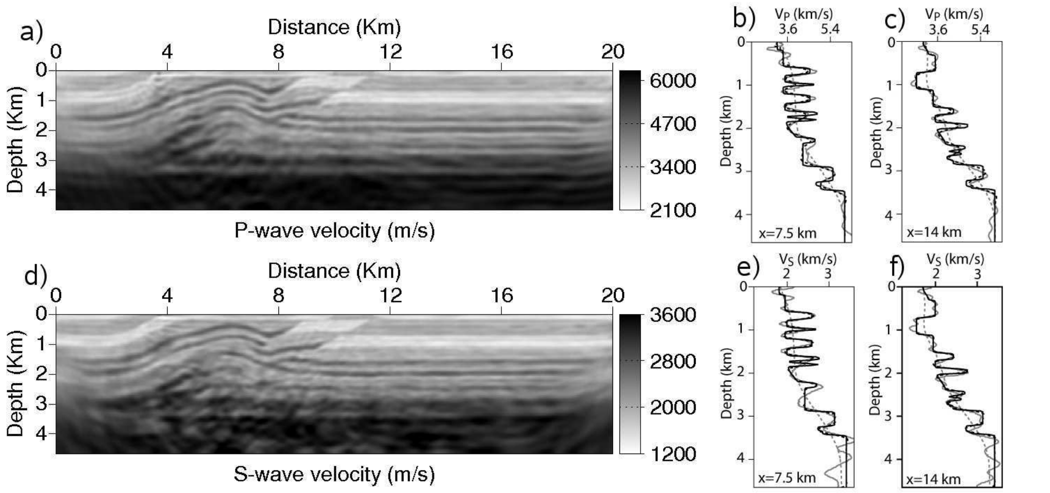

Figure 3: Reconstructed (a) P-wave and (d) S-wave velocity models with preconditioned data and successive frequency inversions. 1D profiles at distance 7.5 km (b)-(e) and 14 km (c)-(f). Profiles of the starting and the true models are plotted with dashed gray lines and solid black lines respectively. A low-pass filtered version of the true model is plotted with a dashed black line for comparison with the FWI results in solid gray lines.

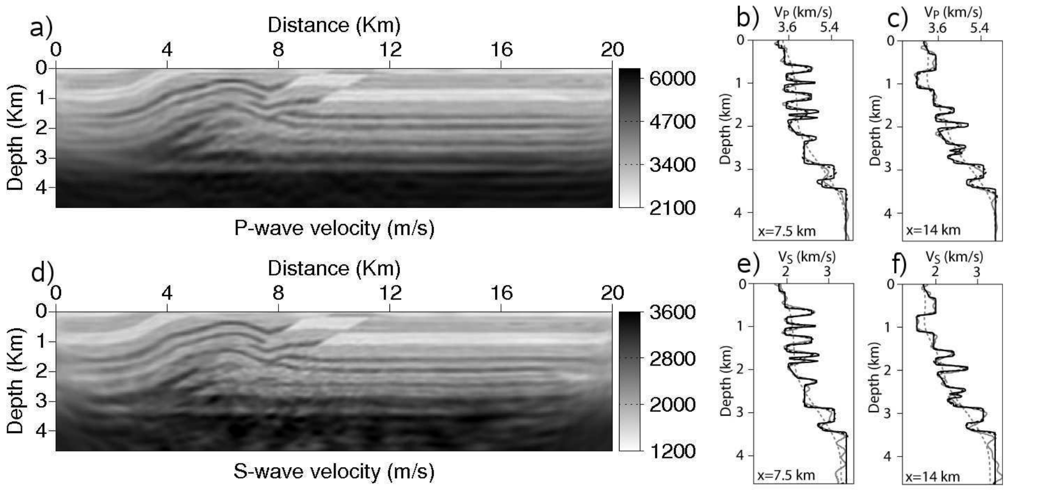

Figure 4: Same than Figure 3 for preconditioned data and simulatenous frequency inversion.

Simultaneous frequency inversions

We assess now another approach based on successive inversions of overlapping frequency groups. By frequency group is meant a set of frequencies which are simultaneously inverted. Summing redundant information from several frequencies allows to better constrain and stabilize inversion while choosing judiciously frequencies to avoid cycle-skipping artefact. Two slightly overlapping frequency groups are successively inverted ([1.7,2.5,3.5] and [3.5,4.7,7.2]). Reconstructed models (Figure 4) are improved compared to that of Figure 3. In particular, final FWI models do not exhibit anymore instabilities in the near-surface. This more accurate representation of the near surface probably contributes to decrease back-propagation of erroneous residuals in the deep part of the model.

L-BFGS versus Preconditionned Conjugate-Gradient

Improved convergence of the L-BFGS optimization with respect to the Preconditioned Conjugate Gradient (PCG) is illustrated in Figure 5 in the case of successive mono-frequency inversion. Convergence curves, shown for the first damping factor of the first and fourth frequencies, clearly show the superiority of the L-BFGS algorithm thanks to an improved non-diagonal estimation of the Hessian matrix.

Conclusion

This study presents an efficient and flexible 2D elastic frequency-domain FWI algorithm with an application to a realistic synthetic onshore model. The application highlights the strong non-linearity of elastic

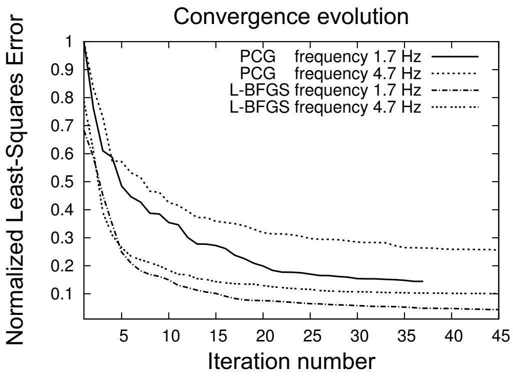

Figure 5: L-BFGS and PCG cost function. Note that L-BFGS accelerates convergence and improves minimization of the objective function.

FWI resulting from the presence of converted and surface waves in the wavefield. Several multiresolution strategies based on different schedules of frequencies and aperture angle selection, implemented with complex-valued frequencies, are tested to mitigate non-linearities of inversion. Tests show that successive inversion of slightly-overlapping frequency groups combined with hierarchical inversions of damped data is critical to mitigate the non linearity of FWI associated with propagation of surface waves in weathered near-surface layers and free-surface reflections. Quasi-Newton L-BFGS method is implemented as optimization algorithm and outperforms PCG thanks to non-diagonal approximation of Hessian matrix with a negligible cost. Future works will focus on starting model building and applications to more complex models including complex topographies and Poisson ratio anomalies before considering applications to real data.

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